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Short Contribution

Least-Squares Estimation of Bottom Topography Using Horizontal Velocity Measurements in the Tsushima/ Korea Straits

NAOKI HIROSE*

Dynamics Simulation Research Center, Research Institute for Applied Mechanics, Kyushu University, Kasuga, Fukuoka 816-8580, Japan

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Several bathymetric data sets are compared and assessed with constraints of an ocean current model and velocity observations. The root-mean-square (rms) differences among the data sets reach 20 m in the shallow Tsushima/Korea Straits. The numerical experiments to simulate the Tsushima Warm Current are performed using four different topography data sets. The JTOPO1 data (MIRC, 2003) give the smallest rms difference to long-term horizontal velocity observations. Several least-squares combinations of the topography data sets are then sought to minimize the rms difference between the observed and modeled barotropic velocities. Most of the data sets reveal a large bias of 30–60 m at the Western Channel compared to independent sounding depths

Keywords:
· Bathymetry,
· Tsushima/Korea Straits,
· Tsushima Warm Current,
· ADCP measurements,
· OGCM,
· inverse estimation.

1. Introduction

The ocean floor is a crucial boundary for the ocean circulations. Although many studies assume the bottom topography to be pre-determined, bathymetric products are still imperfect, as evidenced by the recent discovery of seamounts in the southern oceans by Smith and Sandwell (1994). This study also clarifies the large discrepancies among the data sets in the Tsushima/Korea Straits (TKS).

A land/sea data set of the Earth Topography 5-arc-min grid (ETOPO-5, NGDC, 1988) has long been the only bathymetric data set available as a high-resolution, gridded, digital product. The Digital Bathymetric Data Base-Variable Resolution (DBDB-V, Sandy, 1996) is an updated version of the ETOPO-5, corrected for coastline mismatches.

The bottom topography estimated by Smith and Sandwell (1994, 1997) was a major breakthrough for accurate knowledge of bathymetry. They not only used sounding data reported by ships but also sea surface height measurements given by remote-sensing satellites. The indirect measurements at the surface should reflect the bottom topography, which is solved inversely in their study. The estimated topography was included later in the ETOPO-2 data set (NGDC, 2001).

Korean and Japanese scientists independently developed new bathymetry data sets, SKKU (Choi *et al*., 2002) and JTOPO1 (MIRC, 2003), because of the inaccuracy of other bathymetry data sets in the northwestern Pacific region. The two data sets based on original sources in their countries may be independent from the western products. For example, Book *et al*. (2004) compared the DBDB-V and the SKKU topographies through their tidal modeling and obtained significantly better results with the SKKU.

The present study quantitatively assesses these topography data sets in comparison with ocean current measurements in the TKS. The study region presents shallow bottom relief at 100–200 m depth that seems to directly control the Tsushima Warm Current (TWC). More realistic topography estimates are then be sought as an inverse solution with constraints of an ocean general circulation model and velocity observations.

Recently, Losch and Wunsch (2003) demonstrated that an adjoint method would estimate the bottom topography from sea level observations with an idealized steady barotropic flow. However, difficulties may arise in realistic situations with time-varying, barotropic-baroclinic interactions over complicated geometry.

* E-mail address: hirose@riam.kyushu-u.ac.jp

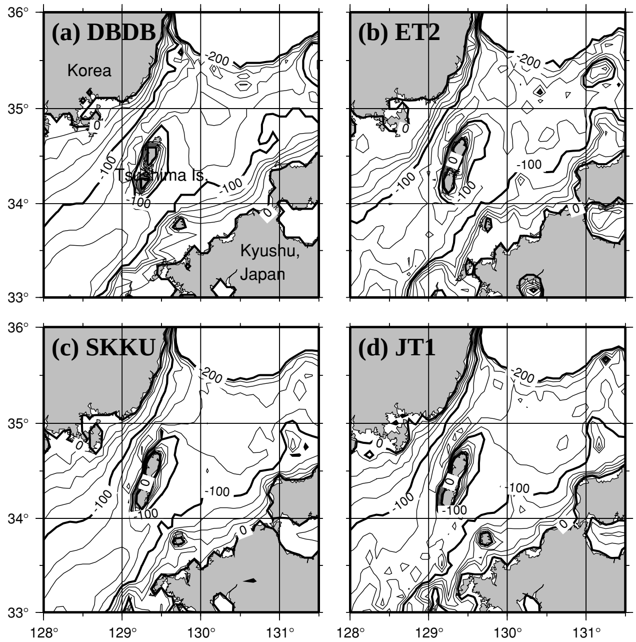

Fig. 1. Bottom topographies of the Tsushima/Korea Straits based on (a) DBDB-V, (b) ETOPO-2, (c) SKKU, and (d) JTOPO1.

Table 1. Root-mean-square (rms) differences among the topography data sets in meter. The differences were taken for water area in the latitude-longitude box of 33–35.5°N and 128–131.5°E with 5-minute resolution. Unit is meters.

(m)	DBDB-V	ETOPO-2	SKKU	JTOPO1
DBDB-V	—	21.67	15.36	18.84
ETOPO-2	21.67	—	20.10	20.22
SKKU	15.36	20.10	—	11.95
JTOPO1	18.84	20.22	11.95	—

For simplicity, this study limits the situation to be near-barotropic, time-mean current, but in a realistic domain. The study has two important advantages for the accurate inverse estimation: a realistic Japan Sea general circulation model (GCM) developed by Lee *et al.* (2003) and long-term velocity measurements with acoustic Doppler current profiler (ADCP) reported by Takikawa *et al.* (2005), which are explained in Sections 2 and 3, respectively. The solutions will be more significant when using the high-resolution model and data. The model results are analyzed in Section 4. The inverse estimation is performed in Section 5, and the results are compared with independent observations in Section 6. The study is summarized in Section 7.

2. TWC Model and the Bottom Topographies

Lee *et al.* (2003) simulated realistic current variations using the Research Institute for Applied Mechanics (RIAM) Ocean Model of the Japan Sea with 1/6° horizontal grid interval. This study focuses on the Tsushima/Korea Straits (TKS), while the model covers the entire Japan Sea with 1/12° grid resolution. The present model adopts non-constant thickness for the bottom cell, known as "partial step topography" (e.g., Adcroft *et al.*, 1997). Since the vertical grid arrangement has a finite interval between 5 m at the surface and 20 m near the bottom in the straits, the partial steps allow milder jumps between the grids and enables smoother barotropic flows.

The model uses climatological monthly averages of wind stress from the European Centre for Medium-Range Weather Forecast, an improved Haney-type formulation for the surface heat flux, and surface salinity restoring. These conditions are the same as those used by Lee *et al.* (2003). Inflow and outflow conditions for the three open straits also follow Lee *et al.* (2003). The TWC flow pattern is insensitive to the inlet velocity conditions: even if all the volume transport starts at the Cheju Strait, the barotropic velocity is hardly altered through the TKS in the model. The bottom friction is parameterized by a quadratic form with a non-dimensional coefficient of $r = 0.01$. The friction parameter will be examined later in terms of difference to the velocity measurements.

Four model experiments have been performed with ocean topography data sets of DBDB-V (Sandy, 1996), ETOPO-2 (Smith and Sandwell, 1997), SKKU (Choi *et al.*, 2002), and JTOPO1 (MIRC, 2003). The traditional ETOPO-5 data are not used in this study because most of the depth values are identical to the DBDB-V except near the coasts and because the DBDB-V coastlines agree better with the World Vector Shorelines (WVS) data set. The root-mean-square (rms) difference between the two is only 8 m in the TKS. Figure 1 shows the isobars drawn from the four topographies. The maps present similar large-scale structures but many small-scale differences are recognized at the deep trench in the western channel, the significant seamount in the ETOPO-2 at the northeastern exit of the straits, the 40–80 m slope west to Kyushu Island, and so on. The fine-scale structures would possibly affect the TWC in terms of its strength and pathway. The data errors may reach 10 to 20% of the depths, judging from the rms differences among the data sets, as shown in Table 1.

3. Horizontal Velocity Measurement

An acoustic Doppler current profiler (VM-BBADCP, 300 kHz, RD Instruments) mounted at the bottom of a commercial ferryboat has been monitoring the horizontal velocity across the TKS six times a week since February 1997. Absolute velocity is obtained directly by tracking the shallow bottom of the straits. Takikawa *et al.* (2005)

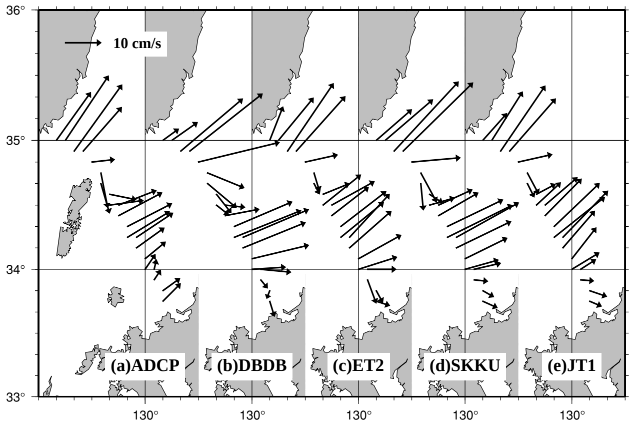

Fig. 2. Mean barotropic velocity distribution by (a) ADCP measurement and by (b)–(e) numerical experiments with several bottom topographies. The averages were taken over 5-year winter and over depth. The model results are shifted to the east by 1°, respectively.

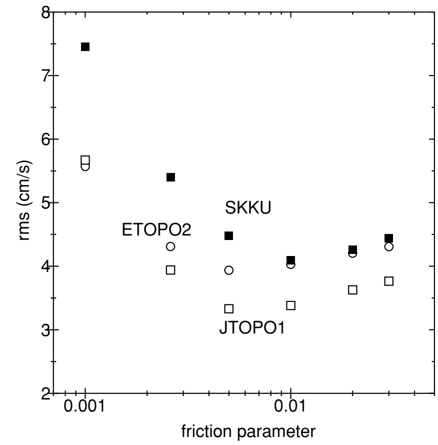

Fig. 3. RMS velocity differences of the simulation and the ADCP measurement as a function of the friction parameter r. The open circles, closed and open squares indicate the experiments with the ETOPO-2, SKKU, and JTOPO1, respectively.

processed the velocity data, eliminating the eight major tidal components of Q_1, O_1, P_1, K_1, N_2, M_2, S_2, and K_2 by harmonic analysis.

Figure 2 shows the horizontal velocity distribution for January to March averaged over the five years from 1998 to 2002 and over depth. The two main streams of the TWC are found in the Eastern and Western Channels of the straits. This study defines the barotropic velocity as a depth-mean flow. The measurement error must be significantly reduced by taking the long-term averages. The winter season was selected for the analysis period to limit the near-barotropic current. Surface cooling mixes the upper ocean in winter, and the vertical density profile becomes highly homogeneous in the shallow straits. The current structure should directly reflect the bathymetric structure upstream and downstream of the measurement line.

The individual beam distance of the ADCP provides an in-situ water depth only: there is no interaction with fluid dynamics. The measured depths of the acoustic beam are only used to validate the estimated bathymetry from the velocity data in Section 6.

4. Simulated Results

The model's output with different topographies is also compared in Fig. 2. The velocity through the TKS reaches a steady annual cycle after an initial half year calculation. The winter averages excluding the initial disturbance are shown in Fig. 2. The simulated velocity with DBDB-V indicates a larger discrepancy than the other three experiments: The near-shore current is separated from the Korean coast, the flow is too strong through the Eastern Channel, and the downstream eddy to the Tsushima Islands is not clear. Similar but weaker differences can be found in the other experiments, too. The rms difference is smallest with the new topography data set of JTOPO1 among the four experiments.

Since the friction coefficient closely relates the bottom topography and the horizontal velocity, ensemble experiments have been carried out with various values of the parameter to determine an optimal value. Figure 3 shows the rms dependence on the friction parameter. The three topographies return minimum rms difference to the ADCP data around the coefficient r = 0.01. The difference diverges for the values smaller than 0.005, and the value r = 0.01 is selected as the near-optimal coefficient.

5. Least-Squares Estimations

The study then seeks optimal combinations of the topography data sets to minimize the rms difference to the measured barotropic velocity. A linear estimation problem may be written as:

$$\mathbf{A}\mathbf{x} \sim \mathbf{y}, \tag{1}$$

where **A** is a set of response functions, **x** is weights to be solved, and **y** is observations. In this study, the winter-mean barotropic velocity of ADCP is used for the observations **y**, and the response functions **A** were already obtained as the model's mean velocity as shown in Fig. 2. The matrix **A** has four columns corresponding to the four topographies of DBDB-V, ETOPO-2, SKKU, and JTOPO1. Since the number of the observation components **y** is larger than the number of the unknown weight

Table 2. Optimal weight coefficients minimizing rms difference between the ADCP measurement and the simulation for several topography data combinations. Also shown are the velocity rms differences (cm/s) in the second left column and in the bottom row.

	rms (cm/s)	E1	E2	E3	E4	E5	E6
DBDB-V	6.34	–0.24	—	—	—	–0.32	—
ETOPO-2	4.05	–0.11	–0.14	—	–0.02	0.47	0.52
SKKU	4.09	0.56	0.27	0.25	—	0.85	0.48
JTOPO1	3.38	0.79	0.87	0.75	1.02	—	—
rms (cm/s)	—	3.16	3.27	3.28	3.38	3.53	3.71

coefficients **x**, this is an overdetermined problem that can be solved by any least-squares method. One more condition,

$$\sum_i x_i = 1 \tag{2}$$

is imposed to conclude the result by a weighted average or a linear interpolation/extrapolation.

This kind of linear analysis would fail if the relationship between the control parameter **x** and the observation **y** is strongly nonlinear. The linearity requirement will be justified by recursive use of the estimated topography in the last section.

The solutions are displayed in Table 2. The first estimate E1 using all four topography data sets gives the smallest rms difference (3.16 cm/s) from the ADCP measurements. The negative coefficients on DBDB-V and ETOPO-2 indicate unimportance or inaccuracy of the two data sets, but the extrapolation contributes to produce the optimal combination. If the negative coefficients are not allowed, the best positive combination E3 is given by an average weighting 0.25 on SKKU and 0.75 on JTOPO1.

The estimated bathymetries E1 and E3 are shown as a function of space in Fig. 4. The results are similar to the JTOPO1 data, reflecting the strong weight. The estimated topographies may be slightly smoother than the JTOPO1, which is attributed to the secondary combination with the SKKU data.

The ETOPO-2 experiment returned a smaller rms difference (4.05 cm/s) than the SKKU (4.09 cm/s), but the weight on ETOPO-2 indicated negative values in the estimates E1 and E2. The reason may originate from the similarity between the ETOPO-2 and JTOPO1 data sets. The JTOPO1 data set was produced by the same methodology as that used by Smith and Sandwell (1997) but with additional Japanese ship-sounding measurements. The estimate E4, which determines the least-squares combination of negligible ETOPO-2 and full JTOPO1, supports the inclusion: only minor information can be derived from the ETOPO-2 against the JTOPO1 data.

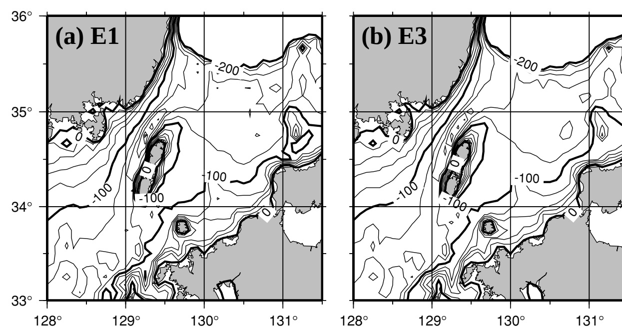

Fig. 4. Bottom topographies estimated by the least-squares combinations (a) E1 and (b) E3.

Unfortunately, the JTOPO1 data are not freely distributed. If the combination is limited to freely accessible data sets, the least-squares combination should be given by E5 or E6. The equivalent weights appearing in E6 indicate independence of the ETOPO-2 and SKKU data, and the expected rms of the combination is significantly smaller than their original rms differences.

6. Independent Data Comparison

The estimated topography E1 is compared with the beam depths measured by the ADCP in Fig. 5. The difference of the E1 from the acoustic depth stays around ±5 m south of 34.7°N (east of 129.6°E), but is larger north of 34.8°N (west of 129.5°E). Most of the topography data reveal depths shallower than the ADCP measurement in the Western Channel, and the least-squares analysis also returns inaccurate solutions.

Book *et al*. (2004) also pointed out the inaccuracy in the deep trough of the Western Channel: the SKKU data provide the depths 60–70 m shallower than the measured values at some mooring sites. Also in the present comparisons, the DBDB-V and SKKU present the depth ~60 m shallower than the ADCP depth in the same trough around 34.9°N, 129.4°E, as shown by Fig. 5.

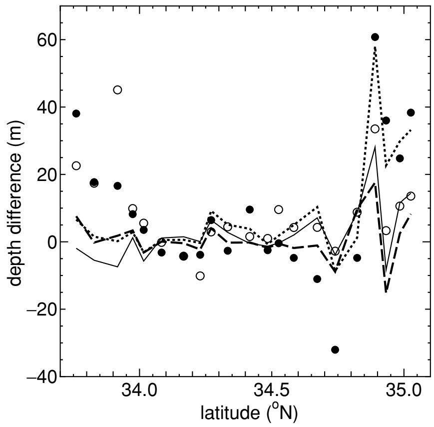

Fig. 5. Difference of depths (in meters) between the gridded topography data and the ADCP beam along the ferry line. Black and white circles indicate the DBDB-V and ETOPO-2, dotted, dashed, and solid curves correspond to the SKKU, JTOPO1, and an optimal combination E1, respectively. The rms differences between the data sets and the beam depths are 21.0, 12.7, 15.4, 6.6, and 8.4 m for the DBDB-V, ETOPO-2, SKKU, JTOPO1, and the estimates E1, respectively.

The rms difference between the estimates E1 and the beam depths is not clearly different from the original topographies. The velocity pattern should reflect the bottom topography upstream and downstream of the observation line and the present inverse estimation is expected to correct the non-local topography at the same time.

7. Discussion and Conclusions

The present study compared the available bathymetric data sets and found a large uncertainty of 10–20 m in spite of the shallow depth in the Tsushima/Korea Straits (TKS). Ensemble experiments with the Tsushima Warm Current (TWC) model were performed with four different topography data sets and with various friction parameters, and the JTOPO1 data (MIRC, 2003) gave a minimum rms difference in comparison with the horizontal velocity measured by a ship-mounted ADCP.

Linear combinations of the available bathymetric data sets were further examined by solving a least-squares problem between the observed velocity and the model outputs. The optimal combination was obtained with strong weights for the JTOPO1 and SKKU data and weak negative ones for the DBDB-V and ETOPO-2. The best positive combination was given by an average weighting of three quarters on the JTOPO1 and one quarter on the SKKU.

Since the linear analyses do not always work well due to nonlinearities of the fluid dynamics, an additional experiment was performed with the estimated topographies E1-E6. The rms differences between the simulated velocity and the ADCP measurement were the same as expected by the inverse estimation. It is concluded that the modelled near-barotropic velocity maintains a strong linear relation with the bottom relief of the TKS.

The optimal topography estimated from the observed velocity should reflect upstream and downstream structures through the inversion process. However, the possible topographic influence may not be too far away from the observation line because changes of the barotropic inflow condition hardly alter the mean velocity pattern at the central part of the TKS. An important future study is to seek the weight solutions as a function of space by including various ocean measurements.

The friction parameter was set to $r = 0.01$ after the sensitivity experiments. Since the minimum rms compared to the ADCP was obtained with coefficients slightly larger and smaller than $r = 0.01$ for the SKKU and JTOPO1 (or ETOPO-2), respectively, as seen in Fig. 3, the optimal parameter would fall around $r = 0.01$.

The results may depend on other factors, such as the model's grid resolutions, surface forcings, or diffusion parameters. At the moment it is very difficult to separate their influences exactly. I can only infer the strong relation between the bathymetry and the barotropic velocity based on the analyses in this study. One more inversion is reported here to check bias effect. The least-squares problem (1) is solely solved without the average condition (2). The results are similar to Table 2 but the overall average depth is ~15% shallower than the original data sets. Real water depths might be shallower than the data sets on average, or the simulated estimates can be biased due to the model's uncertainties. For example, the present GCM requires a minimum of two vertical levels, which are 5 and 7.5 m in this study: Any shallow regions of the model retain 12.5 m depth for numerical reasons. It should be noted that the estimated results still contain such uncertainties, which need to be improved in future.

Many topography data sets have provided depths 30–60 m shallower than the ADCP beam distances in the Western Channel. Accurate depth measurements are to be expected in Korean exclusive economic zone. Advanced estimation methods, such as the Kalman filter or an adjoint method, may be effective to interpolate the individual depth data objectively and dynamically.

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