A New Expression for the Production Rate of Sea Water Droplets on the Sea Surface

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A new set of empirical formulas for the production rate and the number concentration of sea-water droplets on the sea surface are proposed, synthesizing past observation data of sea-salt particles in the sea and water droplets in wind-wave tanks. A new level $z_e$ is introduced as the effective wind-sea surface where sea-water droplets are produced. The new formulas are expressed in linear functions in logarithmic scales of $u^2 / \nu \sigma_p$, a parameter to describe overall conditions of air-sea boundary processes, where $u$ is the friction velocity of air, $\nu$ the kinematic viscosity of air and $\sigma_p$ the peak angular frequency of wind-wave part of wave spectra. A model of coexistence of spray droplets and suspended particles near the sea surface is proposed. As for the independent parameter, a comparison between the uses of $u^2 / \nu \sigma_p$ and of $u^3$ which was the traditional way of parameterization excluding wave measure, shows that the advantage of using $u^2 / \nu \sigma_p$ is statistically significant with a confidence limit 89% in F-test.

1. Introduction

Sea-water droplets produced on the sea surface by wave breaking are expected to contribute to vapor and heat transfer processes from the sea to the atmosphere, through evaporation of water during their suspension in the air before falling back to the sea surface. The production rate of sea-water droplets on the sea surface is thus one of the basic quantities in the local air-sea interaction processes, especially under strong wind and high wave conditions where the wave breaking occurs significantly and more spray droplets are produced.

In earlier ages, Fournier d’Albe (1951) and Moore (1952) observed sea-salt particles at a level in the atmospheric boundary layer above the sea surface. Monahan (1968) made detailed observations close to the sea surface. Toba et al. (1971) made an observation of the particle concentration together with their vertical distribution at a tower station in the sea. Chaen (1971, 1973), Toba and Chaen (1973) observed sea-salt particles at three levels in the Indian Ocean, the East China Sea and the Pacific south of Japan. Preobrazhenskii (1973) and de Leeuw (1986) made measurements at many levels in the North Atlantic.

Experiments in laboratories were reviewed by Wu (1979) but data were not so many. Toba (1961), Koga (1981) and Koga and Toba (1981) measured the distribution of water droplets in wind-wave tanks. Koga and Toba reported a set of data of the number concentration and distribution of water droplets in the vicinity of breaking wind-waves including the region of wave troughs. They also reported that there was a layer of roughly uniform concentration of spray droplets with respect to height in the vicinity of wave surfaces. De Leeuw (1986) found somewhat similar features in the vertical profiles of giant sea-salt particles as measured in the high sea.
De Leeuw (1989) presented a survey of size distribution of droplets observed over the ocean. Koga and Toba (1981) also surveyed the size distribution of droplets measured in laboratories. In Fig. 1(a), a synthesis of Fig. 1 of de Leeuw (1989) and the ocean data by Chaen (1973) is shown. Figure 1(b) is reproduced from Fig. 14 of Koga and Toba (1981) by including original data of Toba (1961). The ordinate $\alpha$ in Fig. 1 is the number concentration of droplets per unit increment in diameter in the same way as in Koga and Toba (1981). The data of these size distributions

![Diagram](image.png)

Fig. 1. Size distribution of sea-salt particles. (a) case of ocean data: synthesis of Fig. 1 of de Leeuw (1989) and data by Chaen (1973). (b) case of laboratory data reproduced from Fig. 14 of Koga and Toba (1981) reinforced by original data of Toba (1961).
consist of the data which were observed at various heights above the sea or above the laboratory water surface.

Wu (1990) proposed forms of parameterizations of concentration and size distribution of droplets near the sea surface, on the basis of a composite result of earlier studies (Monahan, 1968; Preobrazhenskii, 1973; Wu et al., 1984; de Leeuw, 1986). The forms proposed by Wu (1990) include the wind speed as an overall conditional parameter, but no effect of wind waves was examined.

Chaen (1973) proposed a set of empirical formulas for the number concentration and the production rate of sea-salt particles at a level $z_1$, which would represent the effective sea surface, from observed data at some levels taken during cruises in the seas and the ocean. These formulas for the production rate were expressed as a set of linear functions of a nondimensional parameter.
in the logarithmic scales, where $u_*$ is the friction velocity of air, $L$ the significant wave length and $\nu$ is the kinematic viscosity of air.

Toba and Koga (1986) reported that a nondimensional parameter $u_*^2/\nu \sigma_p$ is useful to describe the overall conditions of air-sea boundary processes including the rate of wave breaking, whitecap coverage and droplet production, where $\sigma_p$ is the spectral peak angular frequency of wind waves (we should regard this as the peak angular frequency of wind wave part of spectra of sea waves). As for the estimation of $u_*$ from the wind speed observed on a vessel, a new relationship between the roughness length $z_0$ and $u_*$ has been proposed by including the effect of wind waves explicitly (Toba, 1979; Toba and Koga, 1986; Toba et al., 1990; Ebuchi et al., 1990; Toba and Ebuchi, 1991).

In this paper, a new set of empirical formulas is proposed which gives the production rate and the number concentration of droplets at a newly defined effective level of sea surface, $z_c$, as functions of the nondimensional parameter, $u_*^2/\nu \sigma_p$. This result is the improvement of the empirical formulas given by Chaen (1973), synthesizing the data of Chaen (1973) obtained at sea together with the data of Toba (1961) and Koga and Toba (1981) measured in laboratory, and the results of recent studies on the structure of air-sea boundary layer.

In Section 2, we mainly discuss features of the vertical distribution of sea-salt particles in the sea to examine the way of expression of sea-water droplets near the sea surface. The level $z_c$ is defined as the effective sea surface with respect to sea-water droplet production. In Section 3, the procedure of estimating the number concentration of sea-water droplets at $z_c$ is explained, by using observed data in the sea by Chaen (1973) and the data measured in laboratories by Toba (1961) and Koga and Toba (1981), and new expressions for the production rate and the number concentration at $z_c$ are proposed. In Section 4, the size distribution of sea-water droplets is discussed. In the derivation of the new expressions, two different relationships between $z_0$ and $u_*$, and two different independent parameters for expressing formulas were examined for comparison. A discussion on these points is given in Section 5.

2. Vertical Distribution of Sea-Water Droplets and Proposition of Level $z_c$

2.1 Vertical distribution of sea-water droplets

As for the statistical vertical distribution of sea-salt particles in the atmospheric boundary layer on the sea surface, there is a theory of equilibrium distribution by Toba (1965). It is summarized as follows. Assumptions used were:

(1) A steady state holds with no horizontal gradient in wind speed, the number concentration of particles, etc.
(2) The vertical distributions of wind speed, water vapor pressure and temperature obey the logarithmic law.
(3) The vertical transport of sea-salt particles is carried out by the vertical eddy diffusion and fall with the terminal velocity.
(4) Each droplet instantaneously attains its equilibrium salinity, or equilibrium size, to the vapor pressure and the temperature of the surrounding air. Since the vapor pressure and the temperature are given as functions of height $z$, the terminal velocity and the size are functions of $z$, when the mass of salt contained in a particle is given.
(5) When the relative humidity of air is lower than 75%, sea-salt particles remain as supersaturated saline droplets.
Under the above assumptions, the equation of diffusion of the number concentration of sea-salt particles, $\theta$, is written as follows:

$$\frac{d}{dz}(w\theta) + \frac{d}{dz}\left(D \frac{d\theta}{dz}\right) = 0,$$

(1)

where $D$ is the eddy diffusivity and was expressed by $D = ku_0(z + z_0)$, and where $k$ is the von Karman’s constant and equal to 0.4. Integrating (1) with boundary conditions $\theta = \theta_0$ at $z = 0$ and $\theta = 0$ at $z = \infty$, the solution of Eq. (1) was expressed as follows:

$$\log\left(\frac{\theta}{\theta_0}\right) = -m^2/3U_{10}^{-1} \chi\left(RH_{10}, C_D, z\right),$$

(2)

where $m$ is the mass of salt in a droplet, $U_{10}$ and $RH_{10}$ are the wind speed and the relative humidity at $z = 10$ m, respectively, $C_D$ is the drag coefficient of air, and where $\chi$ is a function of $RH_{10}$, $C_D$ and $z$.

According to Toba’s theory, the vertical distribution of the number concentration of sea-salt particles including any value of mass of salt shows a curve very close to a straight line on a logarithmic diagram and it becomes a straight line when $RH_{10} = 98.23\%$, i.e., when there is no vertical gradient in relative humidity. Comparing the observed gradients, however, of vertical distribution with the theoretical ones for various observed data, such as Toba et al. (1971), Chaen (1971), Toba and Chaen (1973), Chaen (1973) found that the former coincides approximately with the latter calculated for $RH_{10} = 95\%$, though the actual $RH_{10}$ at the time of observations was mostly lower than 95%, 90% in an average, and the range was between 64 and 99%. This suggests that the concentration of sea-salt solution of each particle is not in equilibrium with the relative humidity of surrounding air. Thus, the $RH_{10}$ of 95% was named the effective relative humidity.

To obtain an empirical formula of the production rate $F_1$ of sea-salt particles at a level $z_1$ as an effective sea surface, Chaen (1973) estimated the number concentration $\theta_1$ of sea-salt particles at $z_1$ from the number concentration $\theta_0$ of observed values at $z = 6$ m, under relatively wide range from 2 to 17 ms$^{-1}$ in wind speed, on board the Hakuhu Maru during her cruise KH-70-3 in the East China Sea and the Pacific south of Japan. The details of these derivation are referred to Chaen (1973) and Toba and Chaen (1973). The $\theta_1$ and $F_1$ estimated from the observed data were expressed as a set of linear functions of a nondimensional parameter $u_* L / v$ in logarithmic scales.

In the present study, a basically similar process is used to obtain new expressions for the production rate and the number concentration of sea-water droplets, but the nondimensional parameter as the independent variable, and the value of $C_D$, which is used to estimate $u_*$ from the observed wind speed at some levels, are different from those of Chaen (1973). In Chaen (1973), a constant value of $C_D$ of $1.6 \times 10^{-3}$ was used for a wind speed range from 3 to 10 and some odd ms$^{-1}$.

Goroch et al. (1980), Davidson and Schutz (1983) and Fairall and Davidson (1986) extended Toba’s theory of equilibrium distribution to include the effects of thermal stratification. However, we use Toba’s theory in the following discussion, as the observed data used in this article were obtained under near neutral conditions where the absolute values of the bulk Richardson number were smaller than 0.02, where the bulk Richardson number was expressed as follows:
\[ R_i = \frac{z_{10}g(T_a - T_w)}{(273 + T_a)U_{10}^2}, \]  

where \( z_{10} \) is 10 m above the sea surface, \( T_a \) the air temperature at \( z_{10} \), \( T_w \) the sea surface temperature and \( g \) the acceleration of gravity.

2.2 Effective level representing the sea surface: \( z_c \)

Since the sea-water droplets are produced on the rough sea surface which is complicated by the existence of wind waves accompanied by wave breaking, it is not adequate to adopt the mean sea level itself or the level of the roughness length \( z = z_0 \) very close to it, as the effective level representing the sea surface of the production of sea-water droplets. Hence, an appropriate effective level is necessary to be defined.

In Chaen (1973), an effective height \( z_1 \) as shown in Fig. 2 was introduced with an assumption that sea-salt particles were produced on the wave surface above the mean sea level and diffused into the lowest layer of the atmosphere. By means of a statistical consideration of the sea level, \( z_1 \) was decided as follows. The probability density distribution of the water level along wind waves, \( \eta \), is expressed in general by the Gaussian distribution (Cox and Munk, 1954; Kinsman, 1960). Then, the center of the upper half of this Gaussian distribution was adopted as the effective sea surface \( z_1 \). As the maximum of the water level \( \eta_{\max} \) corresponds to a level where the value of probability density of \( \eta \) is nearly equal to zero, \( z_1 \) was expressed by

\[ z_1 = \frac{0.6745 \eta_{\max}}{3}. \]  

![Schematic representation of the effective sea surface for the sea-water droplets production.](image)

Fig. 2. Schematic representation of the effective sea surface for the sea-water droplets production. (a) \( z_1 \) reproduced from Fig. 30 of Chaen (1973), (b) \( z_c \), a new level adopted in this study.
In general, it is known that $\eta_{\text{max}}$ is nearly equal to the significant wave height $H_s$ (Longuet-Higgins, 1952). Finally, $z_1$ was given by

$$z_1 = 0.6745 H_s / 3.\quad (5)$$

Assuming the wind-wave conditions, $H_s$ can be calculated from the observed significant wave period $T_s$ by using the 3/2-power law between the nondimensional wave height $H^*$ and the nondimensional wave period $T^*$,

$$H^* = BT^{*3/2},\quad (6)$$

where $H^* = gH_s/u_w^2$, $T^* = gT_s/u_w$, and $B = 0.062$ (Toba, 1972).

In contrast to $z_1$, a new effective level $z_c$ adopted in this study is based on the result of experiments by Koga and Toba (1981). They showed that the feature of vertical distribution of water droplets near wind-wave surfaces is divided into two parts by the height, $z_c$, which was given by the crest level of waves of the 1/10 maximum wave height, $H_{1/10}$, or by

$$z_c = H_{1/10} / 2 \approx 0.635 H_s,\quad (7)$$

as shown schematically in Fig. 2. At $z > z_c$, the distribution decreases almost exponentially with the increase of height, while the distribution is approximately uniform at $z < z_c$. According to this feature of the vertical distribution, we adopt $z_c$ as the effective level where sea-water droplets are produced.

3. Production Rate and Number Concentration of Sea-Water Droplets on the Sea Surface

3.1 Estimation of the number concentration at $z_c$ from data observed on board

The observed data on board by Chaen (1973) and Toba and Chaen (1973) are used to obtain the formulas of the number concentration $\theta_c$ of sea-water droplets at $z_c$. Those data include the wind speed at 20 m level, $U_{20}$, the significant wave period $T_s$ for wind waves, the percentage of the whitecap coverage, the number concentration $\theta_w$ of sea-salt particles at $z = 6$ m for $1.0 \leq \log m < 5.0$, where $m$ is the salt mass contained in a particle in $10^{-12}$ g unit, the air temperature at 10 m, $T_a$, and the sea surface temperature $T_w$. We estimate $\theta_c$ by means of the following manner.

(1) Classification of sea-salt particles with respect to the salt mass

According to Toba’s (1965) theory of equilibrium distribution, the vertical distribution of sea-salt particles depends on the contained salt mass $m$. The $m$ is in fact a conservative quantity for each sea-salt (or sea-water) particle. Consequently, the sea-salt particles are classified by $m$ instead of the particle size so that the production rate and the number concentration for each class of $m$ can be estimated. The sea-salt particles observed by Chaen (1973) are divided into 8 classes depending on values of log $m$. The classes are shown in Table 1.

Among data observed during the KH-70-3 Cruise, some data were not used in the present study, since some of them were not in good condition where the 3/2-power law and/or the theory of equilibrium distribution could be applied, and some of the data seemed to be unnatural in the form of size distribution in comparison with the feature shown in Koga and Toba (1981). We have excluded these data from Table 3 of Toba and Chaen (1973) for the estimation of $\theta_c$ from $\theta_w$. The
Table 1. Classification with salt mass, \(m\) (\(10^{-12}\) g unit).

<table>
<thead>
<tr>
<th>class</th>
<th>range of (\log m)</th>
<th>class</th>
<th>range of (\log m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\log m &lt; 1.25)</td>
<td>5</td>
<td>(2.75 &lt; \log m &lt; 3.25)</td>
</tr>
<tr>
<td>2</td>
<td>(1.25 &lt; \log m &lt; 1.75)</td>
<td>6</td>
<td>(3.25 &lt; \log m &lt; 3.75)</td>
</tr>
<tr>
<td>3</td>
<td>(1.75 &lt; \log m &lt; 2.25)</td>
<td>7</td>
<td>(3.75 &lt; \log m &lt; 4.25)</td>
</tr>
<tr>
<td>4</td>
<td>(2.25 &lt; \log m &lt; 2.75)</td>
<td>8</td>
<td>(4.25 &lt; \log m)</td>
</tr>
</tbody>
</table>

Following are the reasons of exclusion with the excluded run numbers in parentheses.

a) The stratification was not near neutral because of large differences between \(T_o\) and \(T_w\) (3, 4, 14).

b) The value of \(T_s\) was not measured or not accurate enough since the values were smaller than 1 s, or the values were for swells (2, 13, 15, 25, 33, 60, 61).

c) The feature of the size distribution was not natural, or there were very few sea-salt particles in classes 1 and 2 (10, 11, 12).

d) The percentage of wave breaking area of wind waves, or the whitecap coverage was zero. In this case, however, only particles larger than class 4 are excluded (16, 22).

Since this exclusion of data results in reducing the number of observed data available, the interval of class of 0.5 in \(\log m\) is adopted, which is twice that of Chaen (1973), in order to increase the number of particles in each class.

(2) Vertical distribution of sea-salt particles at \(z > z_c\)

Toba’s theoretical expression (2) for the vertical distribution of sea-salt particles with 95% of \(RH_{10}\) as the effective relative humidity, is used to estimate \(\theta_c\) from \(\theta_0\). Using \(u_\ast\), instead of \(C_D\) since \(C_D = u_\ast^2 / U_{10}^2\), and substituting 95% into \(RH_{10}\), Eq. (2) is rewritten as

\[
\log(\theta_{20} / \theta_{0n}) = -m_n^{2/3} U_{10}^{-1} \hat{\chi}(u_\ast, z),
\]

where subscript \(n\) denotes the class of \(m, m_n, \) the median of class \(n, \) and \(\hat{\chi} \) is now a function of \(u_\ast\) and \(z\). Eliminating \(\theta_0\) from two equations of Eq. (8) for \(z = 6\) m and \(z = z_c\), we obtain a relation between \(\theta_c\) and \(\theta_6\) as

\[
\log(\theta_{cn} / \theta_{6n}) = -m_n^{2/3} U_{10}^{-1} \left( \hat{\chi}(u_\ast, z_c) - \hat{\chi}(u_\ast, z_6) \right).
\]

(3) Relation of \(z_0\) and \(u_\ast\)

To calculate \(U_{10}\) and \(u_\ast\) in Eq. (9) from \(U_{20}\), we use a new relation of \(z_0\) and \(u_\ast\) proposed by Toba et al. (1990, hereafter we call TIKEJ),

\[
gz_0 / u_\ast^2 = 0.020 \left( \sigma_p u_\ast / g \right)^{-0.5}
\]

Together with the logarithmic wind profile
\[ \frac{U_a}{u_\ast} = \left(\frac{1}{k}\right) \ln \left( \frac{z_a}{z_0} \right), \]

where \( U_a \) is the wind speed at a height \( z_a \).

The peak frequency of wind-waves \( \sigma_p \) is calculated from \( T_s \) by

\[ \sigma_p = 2\pi / 1.05T_s. \]

Eliminating \( z_0 \) in Eqs. (10) and (11), we obtain a nonlinear equation of \( u_\ast \).

\[ u_\ast \ln u_\ast - \left( \frac{2}{3} \right) u_\ast \ln \left( \frac{g\sigma_p}{0.020} \right) + \left( \frac{2}{3} \right) kU_a = 0. \]

The \( u_\ast \) can be obtained as the solution of Eq. (13) by using \( U_{20}, T_s \) and \( z_a = 20 \text{ m} \) which were included in the observed data of the KH-70-3 Cruise.

In summary, the procedure to estimate \( \theta_c \) from \( \theta_6 \) is as follows. First we calculate \( \sigma_p \) using Eq. (12) from the observed \( T_s \). Then, we calculate \( u_\ast, z_0 \) and \( U_{10} \) from Eqs. (13), (10) and (11) by using observed \( U_{20} \) and \( \sigma_p \). The \( z_c \) is calculated from Eq. (7) with \( H_c \), obtained by using the 3/2-power law Eq. (6) from \( T_s \) and \( u_\ast \). Finally, \( \theta_c \) can be obtained from Eq. (9) with observed \( \theta_6 \). This series of calculations is performed for each class of salt mass, \( m \).

The production rate of sea-water droplets at \( z_c \), i.e., \( F_c \), is given by

\[ F_c = w_s \theta_c, \]

where \( w_s \) is the terminal velocity of sea-water droplets at \( z_c \) and it depends on the salt mass \( m \) owing to the assumption (4) of Toba’s theory of equilibrium distribution described in Subsection 2.1.

### 3.2 Expression of production rate and number concentration of sea-water droplets at \( z_c \)

Chaen (1973) proposed a set of linear functions of the nondimensional parameter \( u_\ast L/v \) in the logarithmic scales for the expressions of the production rate \( F_1 \) and the number concentration \( \theta_1 \) of sea-salt particles at \( z_1 \). We intend to express the production rate \( F_c \) and the number concentration \( \theta_c \) at the new effective level of sea surface \( z_c \) as a function of nondimensional parameter \( u_\ast^2/v\sigma_p \). In the case of using \( u_\ast^2/v\sigma_p \) as the parameter, it is expected that \( F_c \) and \( \theta_c \) are proportional to \( u_\ast^2/v\sigma_p \) according to the concept that various values related to the air-sea boundary processes, such as the rate of wind-wave breaking and the percentage of whitecap coverage, are proportional to this parameter as proposed by Toba and Koga (1986). In other words, we might expect a set of linear functions with a gradient equal to unity on the logarithmic diagram.

We have used the following equation to express the relation between \( \theta_c \) and \( u_\ast^2/v\sigma_p \):

\[ \log \theta_c = C_0 \log \left( \frac{u_\ast^2}{v\sigma_p} \right) + C_1. \]

The coefficients of Eq. (15) have been determined for each class of \( m \) by means of the method
Table 2. Values of $C_0$ and $C_1$ in Eq. (15) for number concentration $\theta_c$.

<table>
<thead>
<tr>
<th>class</th>
<th>$C_0$</th>
<th>$C_1$</th>
<th>$C_1$ for $C_0 = 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.79</td>
<td>-3.35</td>
<td>—</td>
</tr>
<tr>
<td>2</td>
<td>0.69</td>
<td>-3.11</td>
<td>—</td>
</tr>
<tr>
<td>3</td>
<td>0.38</td>
<td>-2.71</td>
<td>—</td>
</tr>
<tr>
<td>4</td>
<td>0.67</td>
<td>-4.16</td>
<td>—</td>
</tr>
<tr>
<td>5</td>
<td>0.98</td>
<td>-5.77</td>
<td>-5.84</td>
</tr>
<tr>
<td>6</td>
<td>1.02</td>
<td>-6.20</td>
<td>-6.14</td>
</tr>
<tr>
<td>7</td>
<td>0.80</td>
<td>-5.63</td>
<td>-6.40</td>
</tr>
<tr>
<td>8</td>
<td>0.93</td>
<td>-6.55</td>
<td>-6.88</td>
</tr>
</tbody>
</table>

Fig. 3. Relation of $\hat{\theta}_c$ and $u^2/\nu\sigma_p$ in logarithmic scales for the data of Chaen (1973) together with a straight line of slope $45^\circ$. 
Table 3. Proposed values of coefficients of Eqs. (15) and (17) for ocean data.

<table>
<thead>
<tr>
<th>class</th>
<th>Eq. (15) for $\theta_c$</th>
<th>Eq. (17) for $F_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_0$</td>
<td>$C_1$</td>
</tr>
<tr>
<td>1</td>
<td>0.79</td>
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<td>5</td>
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</tr>
<tr>
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<td>-6.40</td>
</tr>
<tr>
<td>8</td>
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<td>-6.88</td>
</tr>
</tbody>
</table>

of least squares and are shown in Table 2. The values of $C_0$ for classes larger than 4 are nearly equal to 1. Figure 3 shows the slope of $\log \theta_c$ with respect to $\log(u^2/\nu\sigma_p)$ together with a straight line of slope $C_0 = 1.0$, that is, $45^\circ$. Namely, the ordinate $\hat{\theta}_c$ is a normalized value of $\theta_c$ so that

$$\log \hat{\theta}_c = \log \theta_c - C_1,$$  \hspace{1cm} (16)

in order to look at the extent of deviation from the line of slope $45^\circ$. The values of $C_1$ determined by putting $C_0 = 1.0$ are also shown in Table 2.

The above results seem to indicate the following situation. Smaller sea-water droplets have smaller terminal velocities, and consequently some droplets which were ejected in the past were also observed together with those ejected near the time of observation. On the other hand, larger droplets cannot stay longer in the air because of their larger terminal velocities and only those which have been ejected at the observation time were measured. Consequently, the observation that Eq. (15) with $C_0 = 1.0$ is applicable to larger sea-water droplets is consistent with the idea of Koga and Toba (1986).

For the production rate $F_c$, the following Eq. (17) corresponds to Eq. (15) for $\theta_c$.

$$\log F_c = C_0 \log \left( \frac{u^2}{\nu \sigma_p} \right) + C_1.$$  \hspace{1cm} (17)

The coefficients of Eq. (17) can be estimated by the same manner as that for $\theta_c$. The values of $C_0$ are equal to $C_0$ of Eq. (15) because of Eq. (14) and that $w_j$ depends on the salt mass $m$ alone.

Eventually, the new expressions for the number concentration and the production rate of sea-water droplets are given by Eqs. (15) and (17) with the values of their coefficients in Table 3.

3.3 Number concentration of water droplets at $z_c$ measured in wind-wave tanks

Now, the data of Koga and Toba (1981) measured in a wind-wave tank are used to estimate the coefficients of Eq. (15) by using measured $u_*$, $z_0$ and $z_c$ together with $\theta_c$ calculated by the following vertical distribution of droplets.
\[
\ln \theta = -\xi z + \text{const.} \quad \text{for} \quad z > z_c,
\]

which was Eq. (7) of Koga and Toba (1981), an approximate expression for \( \theta \) as a function of \( z \). An average value of \( \xi = 0.23 \) was used in our estimation. In this case, the range of \( \log m \) is from 3.5 to 8.0, some parts of which overlap with classes larger than class 5 of Chaen (1973). Note that the center of each class is, however, shifted to a value larger than that of Chaen (1973) by 0.25 in \( \log m \).

The values of \( C_0 \) of Eq. (15) determined by the method of least squares for each class of \( m \) seem to have no specific tendency such as dependence on the size of droplets as shown in Table 4. The average value of \( C_0 \) is 2.8 with a standard deviation of 0.144. In other words, the slope of \( \log \theta_c \) with respect to \( \log (u^2/v\sigma_p) \) is 2.8 times steeper than the case observed over the sea by Chaen (1973).

Using the data of Toba (1961) as another laboratory data, the coefficients \( C_0 \) and \( C_1 \) of Eq. (15) are also estimated with the same procedure as that for Koga and Toba (1981) except the value of \( \xi \) in Eq. (18). \( \xi \) of 0.14 is now used, which is an average value for Toba’s data. The estimated values of \( C_0 \), also shown in Table 4, seem to have no tendency like the case of Koga and Toba. Its average value is 3.9 and more steep. Toba’s data were measured in a wind-wave tank similar but different from that of Koga and Toba, and included less number concentration of droplets. The conditions were also for values of \( u^2/v\sigma_p \) smaller than those of Koga and Toba.

In order to estimate \( C_0 \) and \( C_1 \) of Eqs. (15) and (17) for the laboratory data, we have made a composite data set of Toba (1961) and Koga and Toba (1981). Figure 4 shows a logarithmic diagram of \( \hat{\theta}_c \) vs. \( u^2/v\sigma_p \) in order to look at the extent of deviation from a straight line with the average slope, just as Fig. 3. The values of \( \hat{\theta}_c \) are grouped into two data sets. One includes the values in a range of \( u^2/v\sigma_p \) smaller than \( 10^4 \) and for Toba’s data. The other includes the values in a range of \( u^2/v\sigma_p \) larger than \( 10^4 \) and for Koga and Toba’s data. The values of \( \hat{\theta}_c \) for Toba’s data are also smoother than those of Koga and Toba, and the values of the two data sets are almost continuous with each other as shown in Fig. 4. That is, these data seem to be complementary to each other as for the distribution with respect to \( u^2/v\sigma_p \). The slope for \( u^2/v\sigma_p < 10^4 \) is \( C_0 = 3.9 \), the average value for Toba’s data, and for \( u^2/v\sigma_p > 10^4 \) is \( C_0 = 2.8 \), the average value for Koga and Toba’s data, as shown in Fig. 4. The final expressions for the number concentration and the

<table>
<thead>
<tr>
<th>the range of ( \log m )</th>
<th>Koga and Toba (1981)</th>
<th>Toba (1961)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( C_0 )</td>
<td>( C_1 )</td>
</tr>
<tr>
<td>3.5 &lt; ( \log m ) &lt; 4.0</td>
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<td>-12.72</td>
</tr>
<tr>
<td>4.0 &lt; ( \log m ) &lt; 4.5</td>
<td>2.52</td>
<td>-12.52</td>
</tr>
<tr>
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<td>2.76</td>
<td>-14.04</td>
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<td>-14.71</td>
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<td>7.5 &lt; ( \log m ) &lt; 8.0</td>
<td>2.79</td>
<td>-15.70</td>
</tr>
</tbody>
</table>
production rate for the laboratory data are given by Eqs. (15) and (17) with the values of their coefficients shown in Table 5.

The data of Chaen (1973) used in this paper were taken at a height 6 m far enough from the sea surface. Namely, the droplets observed are only those which can be regarded as droplets

<table>
<thead>
<tr>
<th>the range of log m</th>
<th>( u*^2/\nu \sigma_p &lt; 10^4 )</th>
<th>( u*^2/\nu \sigma_p &gt; 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>( C_0 )</td>
<td>( C_1 ) for ( \theta_c )</td>
</tr>
<tr>
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<td>-18.82</td>
</tr>
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<td>-18.83</td>
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<tr>
<td>7.5 &lt; log m &lt; 8.0</td>
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</tbody>
</table>

Fig. 4. Relation of \( \hat{\theta}_c \) and \( u*^2/\nu \sigma_p \) in the logarithmic scales for a composite data of Toba (1961) and Koga and Toba (1981). Each point is represented by the average value of \( \hat{\theta}_c \) with standard deviation at each value of \( u*^2/\nu \sigma_p \), together with lines expressed by Eq. (15) with three values of \( C_0 \).
suspended in the atmospheric boundary layer of the effective relative humidity of 95%, after their production on the sea surface, and consequently there are no droplets larger than $\log m > 4.5$.

To the contrary, the data of Toba (1961) and Koga and Toba (1981) were taken at heights within 25 cm above the wind-wave surface and they represent the number concentration of spray droplets, many of which were produced by direct splash of water from the wind-wave crests, and which fell to the water surface immediately after their production. In other words, most of droplets measured were produced very locally, and there were few droplets in the upper stream part, that is, few suspended droplets, since the wind waves developed with the increasing fetch in the wind-wave tanks of fresh water. The sizes of measured droplets were mostly larger than those of Chaen (1973), though the range overlaps. The spray droplets are regarded as corresponding

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Fig. 5. Relation of $\tilde{\Theta}_c$ and $u^2/\nu\sigma_p$ for the overlapping size range of data of Chaen (1973), Toba (1961) and Koga and Toba (1981).
to sea-water droplets which are in equilibrium with the air of relative humidity 98.23% on the sea surface.

The relation of $\theta_c$ with $u_*^2/v\sigma_p$ for the suspended droplets is thus different from that for the spray droplets. In the wind-wave tank, $u_*$ and $\sigma_p$ are much larger, and the wave age is much smaller than those in the sea, though $u_*^2/v\sigma_p$ is in the same range of value. We cannot assume that there is no effect which could not be expressed only by one parameter $u_*^2/v\sigma_p$. However, we will interpret the above situation as follows. There are two series of droplets obeying the different distribution mechanisms: the suspended droplets and the spray droplets. The ocean data of Chaen (1973) represent only the former, whereas the laboratory data of Toba (1961) and Koga and Toba (1981) contain the latter mainly.

Figure 5 shows the $\hat{\theta}_c - u_*^2/v\sigma_p$ relationship for the overlapping range of log $m$ of Chaen’ ocean data and latter two laboratory data, where

$$\log \hat{\theta}_c = \log \theta_c - C_1 \quad (19)$$

with the values of $C_1$ for Chaen’s data in each class of $m$, in order to look at the intersection of the two data sets, ocean and laboratory data. The intersection is at a value of $u_*^2/v\sigma_p \approx 10^4$. It is interpreted that for values of $u_*^2/v\sigma_p$ larger than $10^4$, the spray droplets predominate in number at the level very close to the wave surface, and the production rate of spray droplets is proportional to $(u_*^2/v\sigma_p)^{2.8}$ for the above range of the abscissa. The real production rate and the real number concentration of sea-water droplets are the sum of these two series of droplets.

This interpretation will be further investigated by using the aspect of the size distribution of droplets in the next section.

![Graphs](image)

Fig. 6. Distribution of $C_1$ values with respect to log $m$. 
$C_1$ of Eq. (15) gives real values of $\theta$. Figure 6 shows the distribution of values of $C_1$ with respect to $\log m$. These values are in Table 3 for Chaen’s data larger than class 4 and in Table 5 for Koga and Toba’s data. Two straight lines in Fig. 6 have been determined by means of the method of least squares. Their gradients are $-0.7$ for Chaen’s data and $-0.6$ for Koga and Toba’s data, respectively. A well known relation between the number concentration of aerosols $\theta$ and the radius $r$ proposed by Junge (1963) is expressed as

$$\frac{d(\log \theta)}{d(\log r)} = -3.$$  \hspace{1cm} (20)

Conversion of $r$ in Eq. (20) into salt mass $m$ gives

![Diagram of droplet size distribution](image)

Fig. 7. Schematic picture of droplet size distribution reproduced from Fig. 20 of Koga and Toba (1981). A kink is explained by the coexistence of two different processes of droplets production and spreading which are overlapping with each other in their size range.
Thus the above values of gradients do not seem to deviate much from the Junge distribution.

4. Size Distribution of Sea-Water Droplets near the Sea Surface

In Section 3, we showed that the production rate of droplets has two different trends as a function of $u^2/\nu \sigma_p$, according to the estimates derived from measurements in the ocean and those in the laboratory, as shown in Fig. 5. We gave an interpretation that the sea-water droplets on the

![Diagram showing size distribution of droplets](image)

**Fig. 8.** Size distribution of $\theta_v$ for the data of Chaen (1973) and Koga and Toba (1981). Classifications of $\log m$ for both data are also shown.
sea surface consist of the two series, the suspended droplets and the spray droplets. \( \theta_c \) and \( F_1 \) were expressed proportional to \( u^2/v\sigma_p \) in the former, and to \( (u^2/v\sigma_p)^{2.8} \) in the latter.

This interpretation coincides with the result of the study by using a wind-wave tank by Koga and Toba (1981). Namely, they proposed two spreading processes of water droplets, turbulent diffusion and vertical free motion by ejection. The relation between these processes and the size distribution of water droplets, together with the droplets production mechanism, is shown schematically in Fig. 7, which has been reproduced from Fig. 20 of Koga and Toba. The kink in the size distribution is regarded as reflecting the overlap of different processes which is shown by two dashed lines in Fig. 7. According to this, it is considered that the droplets for smaller values of \( \log m \) are under control of the suspension process, whereas droplets for larger values of \( \log m \) are under control of the splashing process as spray droplets. Since the production rate of the spray droplets is expressed by the 2.8th power of \( u^2/v\sigma_p \), the amount of their existence becomes much larger compared with the feature of increase of that of the suspended droplets. It is noticed that the value of \( \log m \) where the kink appears can vary with \( u^2/v\sigma_p \), though Koga and Toba (1981) found the kink at 4 to 5 of \( \log m \) in their data.

In this context, Fig. 8 shows the size distribution of the number concentration \( \theta_c \) of droplets, which were estimated from the data of Chaen (1973) and Koga and Toba (1981) in Section 3. The general form is coincident with the schematic representation of Fig. 7. For the values of \( u^2/v\sigma_p \) of about \( 10^4 \), where the slope for the both types of droplets intersect with each other in Fig. 5, Chaen’s data on board and the laboratory data by Koga and Toba for the \( \theta_c \) values coincide with each other, that is, the data on board and the laboratory data are continuous in Fig. 8. However, \( \theta_c \) of spray droplets for \( 3 \times 10^4 \) of \( u^2/v\sigma_p \) are naturally high values in Fig. 8. A kink at 3.0 to 3.5 of \( \log m \) in Chaen’s data is also seen for \( u^2/v\sigma_p \) larger than \( 10^4 \) in Fig. 8. This may be an instance that the value of \( \log m \) for the appearance of the kink varies with respect to \( u^2/v\sigma_p \).

5. Discussion

We have proposed the new expressions of number concentration and the production rate of sea-water droplets at the effective sea-surface level \( z_0 \), based on the new formula of wind-stress which was proposed by TIKEJ and which incorporated the state of wind-waves by their spectral peak frequency \( \sigma_p \). In this section, we discuss the use of TIKEJ’s \( z_0 \) formula in comparison with the use of traditional \( z_0 \) formula by Charnock (1955). Also we discuss the difference between the use of \( u^2/v\sigma_p \) and the use of \( u^3 \) as the independent parameter. The latter is a parameter sometimes used for expressing the effect of breaking waves on the mixed layer deepening, as it may express working by the wind, though it does not use any measure of wind waves. More commonly, \( U_{10} \) has been used as the only parameter expressing the wind-wave breaking, or the whitecap coverage and the droplet production. Replacing \( U_{10} \) with \( u_* \), the test of \( u^3 \) here is to examine these traditional way of parameterization without any wave measure.

In Chaen (1973), \( u_* \) and \( z_0 \) were calculated by using a constant \( C_D \) of \( 1.6 \times 10^{-3} \). Also \( u_*L/v \) was adopted as the independent parameter to express \( \theta_c \) and \( F_1 \). The difference between the use of \( u_*L/v \) and of \( u^2/v\sigma_p \) as the independent parameter is not so large in the result actually, since the significant wave measure for wind waves is included for both. Consequently, we examine our result with respect to the difference between the uses of Charnock’s formula and of TIKEJ’s formula in estimating \( u_* \) from \( U_{20} \), and between the uses of \( u^3 \) and \( u^2/v\sigma_p \) as the independent parameter.

Charnock’s formula with respect to \( z_0 \) and \( u_* \) is expressed as
with $\beta$ as a numerical constant. In this comparison, the value of $\beta = 0.0185$ is adopted which was proposed by Wu (1980) based on a review of many experiments (hereafter we call the combination of Eq. (22) with this value of $\beta$ the C-W formula). In this case, the equation of $u_*$ corresponding to Eq. (13) is

$$2u_* \ln u_* - u_* \ln (z_a g / 0.0185) + k U_a = 0.$$  \hspace{1cm} (23)

In Table 6, a comparison between the standard deviations from the lines expressed by the form of Eq. (15) with the values of their coefficients which were estimated for each case, for the use of TIKEJ’s formula and C-W formula. The results obtained by these two formulas did not show any significant difference.

Table 6 also contains a similar comparison for the uses of two parameters, $u_*^2 / v \sigma_p$ and $u_*^3$. We examine the fitness of simply proportional expressions, i.e., Eq. (15) with $C_0 = 1.0$, for the

<table>
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<th>class</th>
<th>$u_*^2 / v \sigma_p$</th>
<th>$u_*^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta_c^T$</td>
<td>$\theta_c^C$</td>
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<tr>
<td>1</td>
<td>0.199</td>
<td>0.197</td>
</tr>
<tr>
<td>2</td>
<td>0.165</td>
<td>0.164</td>
</tr>
<tr>
<td>3</td>
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<td>0.350</td>
</tr>
<tr>
<td>4</td>
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<td>0.224</td>
</tr>
<tr>
<td>5</td>
<td>0.210</td>
<td>0.219</td>
</tr>
<tr>
<td>6</td>
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</tr>
<tr>
<td>7</td>
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<td>0.360</td>
</tr>
<tr>
<td>8</td>
<td>0.340</td>
<td>0.356</td>
</tr>
</tbody>
</table>

Table 7. Variation of standard deviation of residuals for Eqs. (15) and (17) among the uses of a family of TIKEJ’s formula.

<table>
<thead>
<tr>
<th>class</th>
<th>TK0</th>
<th>TK1</th>
<th>TIKEJ</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.201</td>
<td>0.199</td>
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<tr>
<td>2</td>
<td>0.166</td>
<td>0.166</td>
<td>0.165</td>
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<tr>
<td>8</td>
<td>0.329</td>
<td>0.329</td>
<td>0.340</td>
</tr>
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</table>
sea-water droplets larger than class 4, or for the case of those directly related to the local air-sea boundary processes.

The values of standard deviation for the case of \( u^2 \gamma \sigma_p \) are smaller than those for the case of \( u^3 \). The number of observed data in each class is insufficient for testing whether the difference between the uses of two parameters are statistically significant or not. Therefore, we have made up a group which includes all of droplets larger than class 4, by normalizing the data in the same manner as Fig. 3. The number of data in this case becomes 85, and the values of standard deviation become 0.298 for \( u^2 \gamma \sigma_p \) and 0.340 for \( u^3 \). This difference is statistically significant with a confidence limit of 89% in F-test, though not with 95%.

We have also examined some variations of the TIKEJ’s formula in the same manner as we did for the comparison with Charnock’s formula. The variations are the use of Toba and Koga formula \( z_0 \sigma_p / u_0 = \gamma \) with the values of \( \gamma \) of 0.015 and 0.025. These variations were already discussed in TIKEJ, Ebuchi et al. (1990) and Toba and Ebuchi (1991). As a result, it has been proven that the features in the logarithmic diagram of \( \theta_c \) vs. \( u^2 / \gamma \sigma_p \) are similar to one another, and data points move slightly on the diagram. As shown in Table 7, there is very little difference in the values of standard deviations among uses of the family of TIKEJ’s formula. In Table 7, TK0 means the formula with \( \gamma \) of 0.015 and TK1 means the formula with \( \gamma \) of 0.025.

The above results seem to show that the way of expressing the production rate and the number concentration of sea-water droplets by means of \( u^2 / \gamma \sigma_p \) is better than the use of \( u^3 \), as the independent parameter.

6. Conclusion

Synthesizing past observation data of sea-salt particles in the sea and water droplets in wind-wave tunnels together with using the results of more recent studies on the structure of air-sea boundary layer, we have proposed a new set of empirical formulas which give the production rate and the number concentration of sea-water droplets on the sea surface. The result items in this study are summarized as follows.

(1) A new level, \( z_c \), has been proposed as the effective level of the sea surface where sea-water droplets are produced.

(2) About the droplet distribution with respect to the parameter \( u^2 / \gamma \sigma_p \), the following interpretation is given. Two series of sea-water droplets obeying different distribution mechanisms exist: suspended droplets and spray droplets. Near the water surface, the spray droplets predominate for the condition of \( u^2 / \gamma \sigma_p > 10^4 \). The number concentration \( \theta_c \) and the production rate \( F_c \) of droplets at the effective level \( z_c \) can be expressed by straight lines on the logarithmic diagram with \( u^2 / \gamma \sigma_p \) as the abscissa: they are expressed proportional to \( u^2 / \gamma \sigma_p \) for suspended particles for all values of \( u^2 / \gamma \sigma_p \) and \( (u^2 / \gamma \sigma_p)^{2.8} \) for the spray droplets for \( u^2 / \gamma \sigma_p > 10^4 \).

(3) A new set of formulas for \( \theta_c \) and \( F_c \) has been proposed. They are expressed by Eqs. (15) and (17) with the coefficients in Table 3 for the suspended droplets and Table 5 for the spray droplets.

(4) As for the size distribution of sea-water droplets, there is a kink in the distribution curve near \( \log m = 3 \) to 3.5: this corresponds to the coexistence of suspended droplets and spray droplets.

(5) The use of \( u^2 / \gamma \sigma_p \) as the independent parameter is better than the use of \( U_{10} \), or \( u^3 \), in expressing the production rate and the number concentration of sea-water droplets. This is statistically significant with a confidence limit of 89%. It coincides with the idea proposed by
Toba and Koga (1986) that various values related to the air-sea boundary processes are well expressed as a function of $u^2/\nu\sigma_p$.

Acknowledgements

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