Quasi Simultaneous Observations in the Arclength Method of Reduction of Astrometric VLBI Data

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Abstract. We discuss the influence of the transformation matrix between the celestial and the terrestrial systems in the arclength method envisioned to analyse VLBI astrometric observations. In particular we focus on the case of quasi simultaneous observations.

1. Introduction

The classical algorithms of analysis and reduction of VLBI observations (MODEST, NNR-NUVEL1, etc.) adjust simultaneously in a global solution all astrometric and geodetic parameters. Dravskikh et al. (1975a, 1975b, 1978) suggested a new strategy of observation which consists in adopting a new observable: the arclength between a pair of radio sources. Following their guidelines, we continued exploring the feasibility of this new method with the purpose to investigate to which extent it enables to solve for the astrometric parameters (coordinates of the radio sources) exclusively.

The new observable—the length of the arc between a pair of radio sources—is constructed on the basis of the two classical VLBI observables, the delay and the delay rate. We discuss first the case of simultaneous observations of a pair of sources using a radio interferometer, and afterwards we analyze the case of quasi-simultaneous observations of both sources of a pair.

2. The Reference System Adopted

The length of the arc between two objects is independent of the orientation of the reference system adopted. To write the equations of observation we have chosen a reference system whose axes are associated to the directions of the baseline vector \( \mathbf{B} \) and to the vector of angular velocity of rotation of the Earth \( \mathbf{\Omega} \) (Arias, 1990, Arias and De Biasi, 1990, 1991). The x-axis is in the direction of the baseline vector, the
y-axis is perpendicular to the plane defined by the baseline vector and the angular velocity of the Earth. Neglecting plate motions and terrestrial tides, the baseline vector remains fixed in a terrestrial system, while the unit vectors in the direction to the radio source \( \sigma \) and in the direction of the angular velocity of rotation of the Earth remain fixed in a celestial system.

3. The Method: The Cases Analysed

In this analysis we assume that the delay \( \tau \) is due only to the geometry of the problem. As the modules of the vectors \( B \) and \( \Omega \) and the arclength \( A \) are not known, we can adopt a priori values \( B_0 \), \( \Omega_0 \) and \( A_0 \), and include their dimensionless corrections \( \Delta B \), \( \Delta \Omega \) and \( \Delta A \) in the parameters to be adjusted.

We analyse first the case of simultaneous observations of two radio sources, provided that each observing station operates two telescopes.

The arclength is commonly expressed as a function of the direction cosine of the directions \( \sigma_1 \) and \( \sigma_2 \) to the sources of the pair. The classical VLBI observables are the delay \( \tau \) and the delay rate \( \dot{\tau} \). In this method we adopt as observable the length of the arc between two sources, so it is necessary to write the direction cosines as a function of \( \tau \) and \( \dot{\tau} \), and to replace them in the expression of the arclength. Up to the first order in the small corrections \( \Delta B \), \( \Delta \Omega \), \( \Delta A \), the differential expression of the arclength leads, after some algebra, to the linear equation

\[
\sin A_0 \Delta A = C_0 + C_1 \Delta B + C_2 \Delta \Omega,
\]

where \( C_0 \), \( C_1 \), \( C_2 \) are functions of the observables \( \tau \), \( \dot{\tau} \) and of the adopted values \( A_0 \), \( B_0 \) and \( \Omega_0 \), and \( \Delta B \), \( \Delta \Omega \) and \( \Delta A \) are the unknowns.

Adopting standard values for the unknown parameters, we estimated the theoretical precision of the method at the level of 0.0004", value resulting from the magnitudes of the terms neglected in the developments that lead to Eq. (1).

Most VLBI stations have at present one radio telescope, making the strategy of simultaneous observations difficult to apply. We have thus considered the case of quasi simultaneous observations of the sources of a pair. A time interval \( \Delta t \) is elapsed between both observations.

The resulting equation of observation, obtained with the same considerations of the previous case, has the form of Eq. (1), but it includes corrections due to the non simultaneity of the observations of the objects of the pair. The geometry of the problem leads to maximum values of these corrections for an East-West oriented baseline and a source on the equator. For this configuration, we evaluated the terms which contain the corrections for time intervals varying from 30 up to 1 minute. In this step of the analysis, the variation of the transformation matrix between the terrestrial and the celestial systems plays a fundamental role. Even with the shortest interval, the terms including the corrections of non simultaneity cannot be neglected \( \gg 10^{-08} \).
4. Conclusions

We have analysed the arclength method of reduction of VLBI astrometric observations as applied to two strategies of observation: (a) the sources of a pair are simultaneously observed; and (b) the two sources of a pair are observed within a time interval $\Delta t$ (1 minute $< \Delta t < 30$ minutes).

The arbitrary reference system is defined by the directions of the baseline and of the angular velocity of rotation of the Earth. For the case of simultaneity the resulting theoretical precision is at the level of 0.0004". The set of equations of observation is independent of the directions of $B$ and $\Omega$.

The evolution of the terrestrial reference system relative to the celestial reference system does not permit to apply successfully the method when the sources are not simultaneously observed, even for the shortest interval considered.

In a next step we will perform simulations for the case of simultaneous observations, taking into account all the components of the delay (propagation, clock, relativistic effects, source structure).

This method allows to parametrize the unknowns in two arbitrary coordinates, and to evaluate their corrections with respect to adopted a priori values.

REFERENCES