Satellite Position Determination by Difference of Range (DOR) Measurements

Istvan KARDO$\mathrm{\ddot{O}}$S

_Satellite Geodetic Observatory, 2614 Penc, Hungary_

1. Introduction

The Earth satellites are widely used for various scientific investigations. The position and motion of these satellites must be determined accurately enough to satisfy the requirements determined by their scientific goals. To locate a satellite there are two methods:

- geometric, and
- dynamic solutions.

In the case of geometric solution the orbit can be determined from the synchronous measurements of several stations (at least 3), made either for satellites or terrestrial stations. The orbit determination can be made by absolute or relative positioning. Using the DOR measurements we can have an absolute positioning solution, where the accuracy of the satellite orbit depends basically on the stations’ configuration (station coordinate accuracy + geometry), and on the measurement’s accuracy (including the incertitude caused by the receivers and the signal propagation).

In the second case—using a dynamic model—the orbit determination accuracy depends not only on the accuracy of the measurements and on the station configuration, but even on the accuracy of the modelling. In the case of highly elliptic orbit and large satellites the modelling errors are considerable mainly at lower altitudes, near the perigee (LICH$\mathrm{\ddot{E}}$TEN and ESTEFAN, 1990; WU et al., 1991). The geometric method provides useful improvements over the dynamic one where its accuracy is superior of the modelling accuracy. If it is valid only for some parts of the orbit, the combination of two methods will be favoured. The high precision orbit determination can help to improve the dynamic models, and so it gives a considerable contribution to increase our geophysical information. The high precision results of the satellite geodesy have already led to discover new effects. Such an effect for example the temporal variation of $J_2$ coefficient, which was detected by the analysis of LAGEOS laser tracking data. The next largest coefficients of the geopotential are those which are resonant with geosynchronous satellites. To investigate the temporal variation of these coefficients the geosynchronous satellites are appropriate. The motion—basically the longitudinal drift—of these satellites is studied in the COGEO$\mathrm{\ddot{S}}$ project, the work in which scientists of the Satellite Geodetic Observatory of Penc are involved.
2. COGEOS Research Project

The COGEOS project (International Campaign for Optical Observations of Geosynchronous Satellites) has been established by IAG and COSPAR as a CSTG subcomission (NOBIL1, 1987). It’s main task—supported by the European Economic Community—is to detect the time variation of the larger coefficients of the geopotential using optical observations of geosynchronous satellites. During the last years this task had been modified: other satellites, which are “semi-synchronous” (12-hours satellites), have been involved in the observations and the radio electric method (based on DOR measurements)—proposed by the Satellite Geodetic Observatory, Penc—has been accepted for satellite tracking (KARDOS, 1990).

The tasks to be carried out in the frame of COGEOS are the followings:
- optical observations (photographic, CCD),
- laser tracking,
- radio electric tracking of active TV and Meteosat satellites,
- carrying out the data analysis.

Several European observatories and institutes are taking part in this project (CERGA, RGO, ORB, GMS-Pisa, Zimmerwald, Penc, etc.). To develop the new radio electric method and to organize the satellite tracking is the main task for Royal Observatory of Belgium (ORB) collaboration with Satellite Geodetic Observatory, Penc. The first step was to test the feasibility of the proposed technique, which has the advantage of being of limited cost and free from weather conditions.

3. Time Synchronization Using Satellite Measurements

There are several methods to compare the time scales of time services, and in the recent years—thanks to the space-based systems—it can be carried out globally with high accuracy. In our case let us consider the so called “one way” method. By this method the terrestrial stations have receivers only and receive the radio signals transmitted by the satellites. If the signal structure is appropriate and can be decoded by the receivers, then the following measurements can be taken:
- code phase (or pseudo range),
- Doppler shift,
- carrier phase.

The most simple is to measure the code phase that is in our case the arrival time of the TV pulses. Considering two stations, the arrangement is seen on Fig. 1. The synchronization pulse—transmitted by the satellite at epoch \( t_0 \)—is received at the two stations at epoch \( t_A \) and \( t_B \) respectively. \( T_A - T_B \) is the time difference between the time scales of stations, \( \tau_{AB} \) is the differential propagation delay. The satellite is found on a hyperboloid the focal points of which are \( A \) and \( B \). The following relationship can be written:

\[
T_A - T_B = \tau_A - \tau_{AB} - \tau_B.
\]
The measurements for these quantities can be used from two different points of view:

1. If $\tau_{AB}$ is known, the time difference between the two time scales can be determined, and so the time synchronization is accomplished.

2. If $T_A - T_B$ is known, the differential propagation delay, and from it the difference of range (DOR can be calculated)

$$D_{AB} = R_A - R_B = c\tau_{AB}.$$  

Recent developments in time transfer technique may allow the high accuracy clock synchronization. The highest accuracy—sub nanosecond—is achievable by VLBI and satellite laser technique (Veillet et al., 1990). The time synchronization based on GPS time transfer can also reach the nanosecond level of accuracy (Imae et al., 1993). Thus if the two stations are equipped with GPS time receivers, we can consider that $T_A - T_B$ is known and the difference of range can be determined with high accuracy. If more stations are involved in such measurements, then the satellite orbit of synchronous observations can be determined from the DOR measurements by purely geometric method (Kardos and Frey, 1993). From the measurements the range rate can be also calculated:

$$c(t_{Ai+1} - t_{Ai}) = PSR_{i+1} - PSR_i = \Delta PSR.$$  

The range rate accuracy can be higher than the DOR accuracy, since it does not depend on the time scale differences. If the Doppler shift is also measured, the DOR determination accuracy can be increased by the use of a “carrier smoothed range” method.

In the case of geosynchronous satellites the DOR determination can be realized by simple stations, thus this method was proposed for the COGEOs project to measure the longitude drift rate of the satellite.
4. Measurements in the COGEOS Campaigns

The method proposed for the COGEOS is based on the interferometric principle, by measuring at 3 or more stations the arrival time of the synchronous pulses of the TV signal broadcasted by the satellite. The stations were equipped with commercial satellite TV receivers, and high stability time services. To make test measurements tracking campaigns have been organized by ORB in 1991 and in 1993. Altogether 10 European stations have participated in these campaigns. The station configuration is seen on the Fig. 2. In the first campaign different time synchronization methods were used (local TV, satellite TV, traveling clock, GPS receiver). Thus the desired synchronization accuracy (10 ns) was not achievable. During the second and third campaigns all stations were equipped with GPS time receivers: so the delay of the local clock, used for the TV signal measurements, could be determined with respect to GPS. The measurement setup for two stations is shown on Fig. 3. Figures 4 and 5 show the measured time differences and on Fig. 6, a GPS measurement can be seen.

To determine the absolute coordinates of the satellite a geometric (analytic) solution was used. By this analytical solution the satellite coordinates are determined by direct computation from at least 3 independent time differences. If more than 3 time differences are available, i.e. more than 4 stations are involved in the process-

![Map of COGEOS Stations](image)

Fig. 2. Stations participating to COGEOS experiment Brussels, Prague, Cagliari, Torino, Pene, Borowiec, Teddington, Metsahovi, Besancon, San Fernando.
Fig. 3. COGEOS experiment measurement setup.

Fig. 4. Time difference measurements for satellite TV signal. The RMS of 1 Hz and 50 Hz measurements are 20.3 ns and 2.7 ns respectively. Time difference (ns), time (sec).
Fig. 5. Time difference measurements between Penc and Torino stations. Second COGEOS campaign: 23.02.93.–05.03.93.

Fig. 6. GPS time measurements at Prague station. Second COGEOS campaign: 23.02.93.–05.03.93.

ing, then the position of the satellite is computed by adjustment based on least squares. The orbit determined by this geometric method is a “true” orbit, which is free from approximation, and modeling errors. Its accuracy depends basically on the quality of the measured data and on the geometric distribution. In our case this latter
may allow to determine only the $Y$ and $Z$ coordinates with a convenient accuracy. Figures 7 and 8 show the $Y$ and $Z$ coordinates of the satellite computed from the Brussels-Penc-Sanchez Fernando-Cagliari stations’ measurements for one week.

Former studies (Davis et al., 1991; Meyer et al., 1991) discussing the time transfer by geosynchronous satellite have already shown that using TV satellite receivers, a few (3–5) ns accuracy can be achieved in time difference measurements. In situations where observing geometry is poor—for example in the case of geosynchronous satellite—a limited accuracy (30–100 m) is achievable, where the geometry is strong—in the case of low orbits—the accuracy can reach a few meter level. To examine the time variation in longitudinal drift of a geosynchronous satellite—which in our case corresponds, with good approximation, to the motion in $Y$ direction—two stations are needed, located at the same latitude on a baseline, parallel with the orbital plane. In this case the variation of range differences can be written as:

$$DR = 2 \frac{DY(Y_1 - Y_2)}{R_1 + R_2}$$

where $Y_1$ and $Y_2$ are the $Y$ coordinates of two stations, $R_1$ and $R_2$ are the range between the stations and the satellite, $DY$ is the differential variation of the $Y$ coordinate of the

![Graph showing Y coordinates over time for different stations.](image)

**Fig. 7.** EUTELSAT II-F2 $Y$ coordinates determined from measurements at Brussels-Penc-Sanchez Fernando-Cagliari stations. Third COGEOS campaign: 02.08.93–13.08.93.
satellite, $DR$ is the differential range variation caused by $DY$ variation.

This formula gives a good approximation mainly to determine the variation of the amplitude, due to the fact, that the radial variations of a geosynchronous satellite are delayed by $\Pi/2$ to the variations in along track. The $Y$ variations, determined by this method are in good agreement with the satellite motion, computed from the ephemerides. This is shown in Fig. 9 for one week.

If the two stations are separated with 1000 km and the DOR measurement’s accuracy is 1 m, then the along track variation of a geosynchronous satellite can be determined with 40 m accuracy. According to the orbit determination requirements for satellites used in COGEOS experiments—discussed in the basic study for COGEOS (NOBILI, 1987)—the tracking accuracy must be better than 1 arcsec, which corresponds to 200 m at geosynchronous orbit. Thus we can say, that the proposed method—DOR measurements by the use of terrestrial time services—is adequate to use it for geodynamical investigations.

The main merit of this method is not the accuracy—even that the COGEOS requirements are satisfied—but the simplicity of stations and the geometric, absolute orbit determination. The technique is suited to the signal structure of an existing, active satellite. The simplicity of stations will enable to build them in mobile form, thus the station configuration can vary according to the requirements of the geometry. If the satellite has an appropriate signal structure and the carrier frequency
Fig. 9. EUTELSAT II-F2 Y coordinate variations deduced from the Besancon-Penc measurements. Third COGEOS campaign: 02.08.93–13.08.93.

is connected to a high stability frequency etalon, it is possible to measure not only the DOR values but even the Doppler shift in one way mode. Thus the orbit determination accuracy can be improved considerably and the method could be used for satellite tracking where the requirements are stronger than in the case of COGEOS.

5. Conclusion

The satellite orbit determination requirements depend on the scientific goals. To satisfy these requirements several tracking techniques and orbit determination methods are used, so an appropriate solution can be chosen where the accuracy meets the requirements. In the case of synchronous observations the so called “true” orbit can be determined by geometric method. If the observations are not synchronous a “modeled” orbit is generated by dynamic method. There are also solutions that combine the two methods to achieve higher overall accuracy (Lichten and Estefan, 1990; Wu et al., 1991). The common is in these solutions the GPS-based tracking of the satellite having on board GPS receiver. The GPS technique may allow not only the positioning with high accuracy, but even to achieve high precision time synchronization. If the satellite tracking stations—using differential ranging system—are equipped with GPS-based time service, it would allow satellite tracking with high
accuracy without on board GPS receivers. The DOR technique—proposed for the COGEOS—has a wider applicability to other similar missions, where the satellite transmits radio signals. In the case of space VLBI satellites a few meter accuracy can be achieved from only the DOR measurements, and if the data of one way Doppler measurements are also available, this accuracy could be improved considerably. The space VLBI satellite orbits have several perturbation effects due to their big shape, the highly elliptic orbit and the small rocket motor activity to modify the satellite orientation. These pose a series of modeling questions to be discussed. By the use of geometric solution it is possible to eliminate some of these questions. The DOR technique is also appropriate for geometric orbit determination, that can be made from global or local observations. In the first case, having a well-distributed global network of tracking stations all the orbit will be free of modeling errors. In the second case the geometric solution is useful if we make observations near the perigee—to have the “true” orbit for a short arc—where the highest uncertainties with force models are expected. To form a tracking network one can rely on the existing time services or GPS reference stations, providing them with receivers of satellite to be tracked.

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