Chapter 7

COSMIC RAYS IN THE ATMOSPHERE: HIGH-ENERGY PHENOMENA

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7.1 Introduction

Cosmic rays outside the earth's atmosphere are termed primary cosmic rays. These high-energy cosmic rays mainly consist of protons with such minor constituents as bared heavy nuclei. When they are incident into the upper atmosphere, they destroy the atmospheric nuclei by colliding with them to produce nucleons and various mesons. These particles produced in the atmosphere are called secondary cosmic rays, in which many different components are contained. For instance, nucleons and various mesons are usually produced immediately from the direct collisions between the primary cosmic rays and the atmospheric nuclei, while some of them are unstable so that they are transformed to other particles as a result of their decay during flight. These particles are also one of the secondary components of cosmic rays. Since the intensity of the primary cosmic rays decreases exponentially as the result of their interactions of the constituents of the atmosphere, most of the components arriving at ground level consist of the secondary cosmic rays.

In order to classify roughly the components of these cosmic rays, whose flux is about one particle/cm$^2$-min. at ground level, the absorbing layer of lead with a thickness of 10~15 cm is generally used. The component which is almost completely absorbed by this layer is defined as the soft component, while that which penetrates through this layer is called the hard component. The former mainly consists of electrons, positrons and gamma-rays and is sometimes defined as the electromagnetic component. When this component is very high in energy, most of these electrons, positrons and gamma-rays produce the so-called cascade showers, by which the numbers of these three components are rapidly increased as repeating both the bremsstrahlung and electron-positron pair creations. Since the electrons, positrons and photons generated during these processes lose their energies in turn, these showers eventually cease to develop. About three quarters of the secondary cosmic rays observed at the ground are the hard component, most of which are muons. Because the bremsstrahlung loss is inversely proportional to the square of the mass of an incident particle, heavy particles are, in general, able to penetrate through the thick
lead slab and only lose a little of their energy.

Both hadrons and the electromagnetic components are sharply absorbed under the ground or the sea and the secondary components finally consist of high-energy muons and neutrinos. Since neutrinos, which are leptons, have no charge and never interact strongly with other components, they are able to penetrate deeply into the earth. In consequence, they would be found to be isotropic deep underground if observable.

As discussed so far, various components of cosmic rays are found everywhere from the top of the atmosphere to deep underground and cause various phenomena characteristic of each component. For this reason, cosmic ray phenomena have been considered as subjects of research for high-energy particle physics for many years. Many particles such as the positron, muon (μ meson), pion (π meson), Kaon and Λ particle have been discovered in the research on secondary cosmic rays. In this chapter, as regards the secondary cosmic rays, we shall consider an outline of the cosmic ray phenomena which are observed in the atmosphere and underground.

The cosmic ray phenomena and their associated methods of observation are schematically indicated in Fig. 7.1.1, and the elementary particles, which will appear in this chapter, are summarized in Table 7.1.1. The intensity of each component in secondary cosmic rays is indicated in Fig. 7.1.2 as a function of the atmospheric depth. At present, cosmic rays are observed using many different methods in accordance with the difference of our aims, and, consequently, the stations or the locations for cosmic ray observations are selected in favor of these methods. Furthermore, the research technique is distinguished on the basis of our projects; for instances, sometimes we aim to directly observe the elementary processes in the cosmic ray phenomena, while, in other cases, the mean properties of cosmic rays which are deduced statistically are searched for by using the observed results on the behavior of each component of cosmic rays. Cosmic ray phenomena are particularly interesting since their energy range is very wide and also their occurrence frequency is highly dependent on the energy of cosmic rays. For example, the primary cosmic rays of energy $10^{15}$ eV, whose intensity is about $10^{-6}$ m$^{-2}$·h·sr, can be detected by means of small sensitive equipment, while the observing method of the cosmic rays of energy $10^{20}$ eV is completely different, since these cosmic rays are observable once a year in an area of 100 km$^2$. Namely, such ultra-high-energy cosmic rays are detected by observing them as the extensive air showers. In fact, to observe these showers is the only method to study such ultra-high-energy cosmic rays. Since the incident flux of cosmic rays is more than $10^{10}$ times different between the low and high energies mentioned above, a question necessarily arises on whether the nature of the primary cosmic rays is the same throughout the whole energy range. Cosmic ray phenomena are complex and entangled, so it is necessary to analyze them by separating them into individual solvable subjects. This may be said to be the goal of cosmic ray research.

The history of the observations of cosmic rays stretches back to the early 1910s. The radiation intensity of cosmic rays was first observed by using the gold-leaf electroscope and later on many different measuring apparata were devised: they are the ion-chamber, the photographic emulsions, the Wilson chamber, the magnetic
cloud chamber, the GM counter, the proportional counter, the nuclear emulsions, the high-pressure cloud chamber, the scintillator, the Čerenkov detector, the emulsion chamber, the BF₃ counter, the neon hodoscope, the discharge chamber, the transition meter and the drift chamber. It should be remarked that many of these apparata were developed specifically to make cosmic ray observations. In accordance with our aim, some of these counters and the range detectors are combined to observe some properties of cosmic rays and the large-scale detecting systems are being operated at present. At the present moment, the scaling-up of these observing systems are being investigated by making use of the conditions available in nature in skillful ways; for instance, the observing system called the fly’s eye to detect the atmospheric Čerenkov lights emitted from air shower particles developing in the atmosphere, and the DUMAND consisting of many photomultipliers to detect the Čerenkov lights emitted by charged particles under the sea.
Table 7.1.1. Classification of fundamental particles.

<table>
<thead>
<tr>
<th>Class</th>
<th>Particle</th>
<th>Mass (MeV)</th>
<th>Charge e</th>
<th>Life (sec)</th>
<th>Decay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photon</td>
<td>$\gamma$</td>
<td>0</td>
<td>0</td>
<td>$\infty$</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>$e^\pm$</td>
<td>0.51</td>
<td>$\pm 1$</td>
<td>$\infty$</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>$\nu_e\bar{\nu}_e$</td>
<td>0</td>
<td>0</td>
<td>$\infty$</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>$\mu^\pm$</td>
<td>106</td>
<td>$\pm 1$</td>
<td>$2.2 \times 10^{-6}$</td>
<td>$e^\pm \nu_e \bar{\nu}_e$</td>
</tr>
<tr>
<td></td>
<td>$\nu_\mu \bar{\nu}_\mu$</td>
<td>0</td>
<td>0</td>
<td>$\infty$</td>
<td>No</td>
</tr>
<tr>
<td>Lepton</td>
<td>$\pi^\pm$</td>
<td>140</td>
<td>$\pm 1$</td>
<td>$2.610^{-6}$</td>
<td>$\mu^ \pm \nu_\mu$</td>
</tr>
<tr>
<td>Hadron</td>
<td>$\pi^0$</td>
<td>135</td>
<td>0</td>
<td>$10^{-16}$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td></td>
<td>K$^\pm$</td>
<td>494</td>
<td>$\pm 1$</td>
<td>$1.210^{-6}$</td>
<td>($\mu^\pm \nu_\mu$)</td>
</tr>
<tr>
<td></td>
<td>K$^0$</td>
<td>498</td>
<td>0</td>
<td>$0.910^{-10}$</td>
<td>($\pi^+ \pi^-$)</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>938</td>
<td>+1</td>
<td>$\infty$</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>n</td>
<td>940</td>
<td>0</td>
<td>940</td>
<td>$\mu e^- \bar{\nu}_e$</td>
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<tr>
<td></td>
<td>$\chi^0$</td>
<td>1116</td>
<td>0</td>
<td>$2.510^{-16}$</td>
<td>$\mu e^- n\pi^0$</td>
</tr>
</tbody>
</table>

Remark ( ) indicates an example.

7.2 Energy Spectra of the Primary Cosmic Rays

The energy spectra of the primary cosmic rays have been observed by means of different methods in accordance with the different energy ranges of these cosmic rays. Those for the cosmic rays of energy between 1 and 15 GeV are derived from the geomagnetic latitude effect on the cosmic rays. The cut-off momenta of the primary cosmic rays vertically incident onto the earth are calculated as zero at the geomagnetic poles and 15 GeV/c at the geomagnetic equator. This cut-off momentum of the incident cosmic rays is also calculated as about 10 GeV/c at 25° north geomagnetic latitude. Since these momenta correspond to the primary protons in the incident cosmic rays, those for $\alpha$-particles and Fe-nuclei are different from the ones described above. In fact, those for these nuclei are a factor 1/2 less than those for protons, since the ratios of their charge to mass numbers $(Z/A)$ are equal or almost equal to 1/2. In order to derive the total energy spectrum for each nucleus in primary cosmic rays, it is only necessary to multiply this spectrum with the mass number of the nuclei under consideration.

The energy spectra of the primary cosmic rays in the energy range from $\sim$ 10 GeV to $\sim$ 10 TeV are estimated from the analyses of the jet showers as observed with the
nuclear emulsions flown on board balloons. In the energy range between ~1 TeV and ~100 TeV, the energy spectra can be estimated based on the theoretical calculations on the generation mechanism of the observed energy spectra for the secondary muons. Furthermore, the energy spectra of the primary cosmic rays in the energy range from ~100 TeV up to ~10^{20} eV are derived from the energies of the primary particles estimated from the analyses of the observed results of extensive air showers. In this regard, the energy spectra of the primary cosmic rays are considered to be the condensed results of the extensive research on cosmic ray phenomena in the earth's atmosphere.

Figure 7.2.1 shows both the energy spectrum of primary cosmic rays in the whole energy range from ~10^5 eV to ~10^{20} eV and that for the proton component in the primary cosmic rays. The integral energy spectrum of the primary cosmic rays derived numerically from the result shown in this figure is expressed as
Fig. 7.2.1. The energy spectra of the primary cosmic rays. $R_p$ and $G_T$ are, respectively, the observed proton spectrum by Ryan\cite{2} and the total spectrum by Grigorov.\cite{1} EAS data are taken from the air shower observation at Yakutsk.

\[ I(>E) = 1.0 \times (E + 2)^{-1.55 - 0.02 \log_{10} E} \text{ cm}^{-2} \cdot \text{sec}^{-1} \cdot \text{sr}^{-1}. \]  \hspace{1cm} (7.2.1)

On the other hand, this spectrum for the proton component is expressed likewise as

\[ I_p(>E) = 0.5 \times (E + 1)^{-1.55 - 0.02 \log_{10} E} \text{ cm}^{-2} \cdot \text{sec}^{-1} \cdot \text{sr}^{-1}, \]  \hspace{1cm} (7.2.2)

where $E$ is the primary energy and is given in units of GeV in these two formulae. These spectra over such a wide energy range as shown above are not necessarily highly exact, but are very convenient to take a glance at the gross nature of the primary cosmic rays, though it is necessary to further collect more data on the energy distribution of these cosmic rays.

When the energy spectrum in some limited energy range is expressed in the form $AE^{-\gamma}$, the index $\gamma$ is obtained as

\[ -\gamma = \frac{\partial \ln I(>E)}{\partial \ln E} = -1.55 - 0.04 \log_{10} E. \]  \hspace{1cm} (7.2.3)
It thus follows that the value of $\gamma$ increases by 0.04 everytime the energy is increased by a factor of 10.

Although the relative abundances of the primary cosmic rays have not been known with respect to the whole energy range as cited earlier, the measured results of these cosmic rays in the relatively lower energies\(^3\) indicate that, when compared with the energy per nucleon, 94.5% of the total number of primary cosmic rays are protons, while the helium component is about five percent. Then, only about 0.5 per cent consists of the nuclei heavier than lithium, but most of them are the medium nuclei such as C, N and O and Fe. If all the nuclei contained in primary cosmic rays were reduced to protons and neutrons, the primary cosmic rays as a whole would be considered as the composite of 89% protons and 11% neutrons. Since the total energy of each nucleus in cosmic rays is given by the product of the energy of a nucleon with the mass number $A$, it follows that the total intensity of the heavier nuclei becomes of the order of that of the protons. Thus, the energy spectrum for all of the primary components become about twice that of the proton component. Whether or not these abundances are extended to those in the higher energy region is not known because of the lack of enough observed data to examine this question.

According to some experimental results,\(^4\) the index of the energy spectrum for the iron component of the primary cosmic rays is rather small compared to those for the other components. If this is the case, the relative abundance of the iron component tends to become higher with particle energy; for instance, the flux of the iron nuclei is about 60% of the total flux in the energy around $10^{15}$ eV. However, it seems that the iron flux sharply drops in the energy region higher than $10^{16}$ eV. The variability of the energy spectrum for the different components of the primary cosmic rays has not yet been clearly studied.

### 7.3 Attenuation of Cosmic Rays in the Atmosphere

The cross-section of collisions between high-energy protons and nuclei at rest is roughly expressed as

$$\sigma = \sigma_0 \left( A^{1/3} + \alpha \right)^2,$$  \hspace{1cm} (7.3.1)

where $\sigma_0 = 4 \times 10^{-26}$ cm\(^2\) and $\alpha = 0.2$ or so. The mean-free path between successive collisions is thus given by

$$\lambda = \frac{A}{N\sigma} \text{ g/cm}^2.$$  \hspace{1cm} (7.3.2)

While high-energy cosmic ray protons are in motion in the atmosphere, their collisions with atmospheric nuclei occur once in a length given by $\lambda = 90$ g/cm\(^2\). Since the cross-section of such collisions tends to increase as given by

$$\sigma' = \sigma (1 + \delta \ln E) = \sigma E^\delta,$$  \hspace{1cm} (7.3.3)
the mean-free path tends to become shorter. In the above Eq. (7.3.3), the energy $E$ is measured in units of TeV and the magnitude of $\delta$ is $0.01 \sim 0.02$\textsuperscript{3).}

The hadrons in the energy range $1 \text{ GeV} \sim 100 \text{ GeV}$ mainly consist of protons and neutrons in the atmosphere and collide with the atmospheric nuclei in a mean-free path of about $90 \text{ g/cm}^2$. In these hadrons, the numbers of protons and neutrons are almost equal to each other. The intensities of these nuclear components in the atmosphere decrease with depth in atmosphere as expressed by

$$I(>E) = I_0(>E) e^{-\lambda}.$$  
(7.3.4)

Thus, the absorption mean-free path $\lambda$ is calculated by using the mean-free path defined as $L$ and the surviving rate of these hadrons as related to the inelasticity $K$ during the collisions. Namely,

$$\frac{dI(>E)}{dt} = - \frac{I(>E)}{L} + \frac{1}{L} I \left( \frac{E}{1-K} \right)$$  \hspace{1cm} (7.3.5)

and

$$\lambda = \frac{L}{1 - (1-K)^\gamma}.$$  \hspace{1cm} (7.3.6)

When we take $L = 90 \text{ g/cm}^2$, $K = 0.5$ and $\gamma = 1.7$, the path $\lambda$ is estimated to be $130 \text{ g/cm}^2$. In fact, this mean-free path $\lambda$ for the nucleons is of the order of $125 \text{ g/cm}^2$ in the atmosphere.

### 7.4 Low-Energy Nucleonic Components

The energy spectrum of cosmic rays in the energy range lower than 1 GeV is the composite of the two components: the nucleon components formed after the nucleons originally in the middle and low energies lose their energy in the atmosphere and the protons and neutrons in the energy range from several ten to several hundred MeV knocked out of the atmospheric nuclei during their collisions with the nucleons initially produced in the atmosphere. The nucleon components of energy higher than 100 MeV lose their energy by the nuclear fragmentations called "stars", the name of which has been given based on the phenomenological shape when many protons and neutrons diverge from one point. In the energy range between $\sim 10 \text{ MeV}$ up to $100 \text{ MeV}$, protons rapidly lose their energy due to ionization losses and are then eventually absorbed, while neutrons lose their energy by collisions with the atmospheric nuclei producing nuclear reactions such as $^{14}\text{N} (n,p)^{14}\text{C}$. The neutrons of energy less than $1 \text{ MeV}$ lose their energy through repeated elastic collisions with the atmospheric nuclei and eventually are absorbed by these nuclei after becoming thermalized.

The geomagnetic latitude effect on all of the components in the secondary cosmic
rays is largest on the neutron component, since the intensity of the primary cosmic rays is much more intense at lower energies. In fact, the intensity of this component in the high latitude region is about three times higher than in the low latitude region. Since the absorption mean-free path for the neutron component in the atmosphere is estimated to be about 150 g/cm² in the high latitude region and about 200 g/cm² in the low latitude region, the difference in the neutron intensity as mentioned above tends to become smaller with atmospheric depth.

The result shown in Fig. 7.4.1 is the energy spectrum for the neutron component obtained from observations at the Mt Norikura Observatory (25°N geomagnetic latitude, 2770 m above sea level and atmospheric pressure 760 g/cm²). Because the neutron energy ranges from 1 eV to 10 GeV, some caution should be considered in comparing the neutron intensities at different energies. In fact, the energy spectrum for the neutrons between 1 MeV and 10 GeV is given as

\[ 4.8 \times 10^{-1} E^{-1.07} \log E \text{ cm}^{-2} \text{ sec}^{-1} \text{ MeV}^{-1}, \]  

(7.4.1)

and is independent of the direction of the observations, but, in Fig. 7.4.1, only the neutron intensity at vertical incidence is shown. In fact, the incident directions of neutrons are isotropic for energies around 1 MeV, while these directions have a zenith angle distribution as \( \cos^2 \theta \) at energies around 10 GeV, where \( \theta \) is the zenith angle. Since the speed of neutrons depends on their energy, the intensity of these neutrons is defined as the product of the density with the speed of the neutrons. In the energy range less than 1 MeV, the deceleration by elastic collisions is the main cause of neutron damping and, furthermore, the cross-section of these collisions changes from

![Graph](image-url)

Fig. 7.4.1. The energy spectrum for the neutron component on the ground (760 g/cm²).
18 g/cm$^2$ to 4.5 g/cm$^2$ at about 100 keV. In consequence, the deceleration becomes more significant and the neutron intensity is decreased by $1/4$. The neutron intensity in the energy range less than 1 keV is thus expressed as

$$\rho\nu = A \frac{1}{E} \exp \left( -15.5 \sqrt{ \frac{E_0}{E} } \right) \, dE,$$  \hspace{1cm} (7.4.2)

where $E_0 = 6.25 \times 10^{-4}$ eV. By multiplying the observed result shown in Fig. 7.4.1 with $4\pi$, since the neutron intensity is isotropic in this low-energy range and by integrating this intensity multiplied with the probability $1/\nu$ for neutrons to be absorbed by the atmospheric nuclei, we obtain the total intensity of the neutrons which can be compared with the observed intensity taken by the BF$_3$ counters. The observed results obtained by the BF$_3$ counters roughly consist of two distinct components, the magnitudes of which are numerically different by a factor of two, but the energy spectrum shown in Fig. 7.4.1 is consistent with that for the lower curve. In this figure, the neutron intensity at 1 eV calculated from the measurements by Yuan$^7$ is plotted on the energy spectrum measured at 50° north geomagnetic latitude. While thermal neutrons wander isotropically in the atmosphere with moving radii of about several ten meters, most of them are absorbed by the atmospheric nuclei before they decay naturally because their life time is as long as about 15 minutes.

The energy spectrum for the neutron component as shown in Fig. 7.4.1 is derived based on the measurements of recoiled protons of energy between 1 and 20 MeV produced from the collisions of the neutrons with hydrogen in the high-pressure cloud chamber at 100 ATM. When we observe these protons with this chamber filled with nitrogen gas, we are able to detect the tracks of protons produced from the star phenomena like the cosmic ray stars in the atmosphere.$^8$) The energy spectrum for these protons is expressed as

$$A \frac{E}{T^2} \, e^{-E/T},$$  \hspace{1cm} (7.4.3)

where $T \sim 2.6$ MeV. The proton energy $E$ used here is not the real proton energy, but is the “equivalent” energy derived by subtracting the Coulomb potential of the nitrogen nucleus from the observed energy of the recoiled proton. The energy distribution of the proton component seems applicable to that of the neutron component, which is now considered as the source of the low energy neutrons in the atmosphere.

### 7.5 High-Energy Nuclear Interactions

When a high-energy nucleon in the energy range from $\sim$100 GeV to $\sim$100 TeV collides with a nucleus at rest, many mesons are produced simultaneously. This phenomenon, known as the multiple meson production, is called a hard shower since the hard component of secondary cosmic rays is produced, and often also called a jet shower as viewed from the morphology of this phenomenon. Until recently, the
research on nuclear interactions in such a high energy region was considered as most
important in cosmic ray research, but the experimental research in this energy region
is now being done by using the collider-type accelerators, in particular, because of the
increase in energy attained by accelerators.

7.5.1 Analysis of the experimental data

In the case of the collision between two protons, the secondary particles released
from this collision into the $\theta^*$ direction in the center-of-mass reference frame are
concentrated forwards into the $\theta$ direction in the laboratory frame to form the pattern
like a jet (see Chapter 3).

The Lorentz transformation between these two frames is given as

$$\tan \theta = \frac{1}{\gamma_c} \frac{\sin \theta^*}{\cos \theta^* + (\beta^*/\beta_c)}, \quad (7.5.1)$$

where $\beta^*$ is the proton velocity in the center-of-mass frame, and $\beta_c$ and $\gamma_c$ are the
proton velocity and the Lorentz factor in the center-of-mass frame as referred to the
laboratory frame, respectively. When both $\beta^*$ and $\beta_c$ are close to unity ($\beta = v/c$), the
above equation is reduced to

$$\tan \theta = \frac{1}{\gamma_c} \tan \frac{\theta^*}{2}. \quad (7.5.2)$$

Thus, the plane $\theta^* = \pi/2$ is seen as the sharp circular cone since $\tan \theta = 1/\gamma_c$.

In the case where the secondary particles are symmetrically released into both
forward and backward directions in the center-of-mass frame, the angle of the cone,
inside which half of the total number of particles released are contained in the
laboratory frame when counting these particles with respect to this angle $\theta$, just
corresponds to the angle equal to that given by Eq. (7.5.2). Thus, this angle denoted as
$\theta_{1/2}$ is called the half angle. By this notion, it follows that $\tan \theta_{1/2} = 1/\gamma_c$ and
$E_p = 2 \gamma^2_c m_p c^2$. Thus, the energy of the incident proton is estimated.

Taking the logarithm of both sides of Eq. (7.5.2), we obtain

$$\log(\tan \theta) = \log \left( \tan \frac{\theta^*}{2} \right) - \log \gamma_c. \quad (7.5.3)$$

In the region where both $\theta$ and $\theta^*$ are very small, the distributions of the secondary

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Fig. 7.5.1. The laboratory and the center-of-mass systems.
particles with respect to $\theta$ and $\theta^*$ are similar to each other as regards $\log (\tan \theta)$ except for the difference at the frame origin.

Defining the momenta in the axial and the transverse directions by $P_\parallel$ and $P_t$, respectively, Equation (7.5.3) is rewritten as

$$\log(\tan \theta) = \log \frac{P_t}{P_\parallel}. \quad (7.5.4)$$

The momentum $P_t$ in the lateral direction is nearly independent of the energy of the incident nucleon, and shows a distribution with a nearly constant mean value. Since this momentum also remains constant for the Lorentz transformation in the axial direction, the distribution for $\log (\tan \theta)$ itself gives that of $-\log P_\parallel$.

The variable ($Y$) often used recently, defined as the velocity, is given as

$$Y = \frac{1}{2} \ln \frac{E + P_\parallel}{E - P_\parallel} \approx \ln \frac{2P_\parallel}{P_t + mc^2} = \ln P_\parallel + \text{const.} \quad (7.5.5)$$

Therefore, this variable $Y$ has properties similar to those of $\log (\tan \theta)$, though its scale is different.

7.5.2 Model of multiple meson productions

The models so far proposed on the multiple meson productions do not necessarily have supporting evidence from particle physics. The dependence on the incident energy of the numbers of particles produced from those models would be summarized as follows: in the vortex model by Heisenberg,\(^9\) which considers the production of low-energy particles, $n \propto E^{1/2}$, where $n$ is the number of particles produced. In the L.O.W. model where secondary particles are radially emitted,\(^10\) $n \propto E^{1/3}$, in the blackbody type model by Fermi,\(^11\) $n \propto E^{3/4}$. This relation is the same as that proposed by Landau\(^12\) in the eruptive fluid model. Furthermore, the fire-ball model\(^13\) as put forward by Niu and Cocconi gives the relation $n \propto E^{3/4}$. These models have been used in order to interpret the properties of the jet showers when they were proposed. The suitability of these models to the experimental results is, of course, dependent on the energy of the incident particles. Furthermore, the experimental results are almost proportional to $E^{1/2}$ in the low energy range as shown in Fig. 7.5.2, but their energy dependence gradually becomes relaxed with the increase of incident energy. The results obtained from direct observations, however, have not been extended beyond $10^{15}$ eV, and the results in the energy range higher than $10^{15}$ eV have been indirectly obtained from the observations of extensive air showers. In other words, no clear-cut result has been obtained so far in such a high-energy range.

At the present moment, the limiting fragmentation and the scaling law of Feynmann\(^14\) are often applied to explain the nuclear interactions in the high-energy region. The first limiting fragmentation is considered as the model that, when an incident nucleon collides with a target nucleus, the secondary particles of high energy are released forward as if the incident nucleon were splitting with no relation to the
kinds of target nuclei. In consequence, this fragmentation plays an important role in the phenomenology of cosmic rays, because the shape of the energy spectrum for the primary cosmic rays sharply changes with particle energy. On the contrary, the production of the secondary particles which occurs isotropically around the center of the center-of-mass frame is sometimes called pionization, though the energy of these particles is not high. Although these two processes are clearly distinguished from each other, they are useful for the understanding of the models as described earlier in this section.

In the scaling law of Feynmann, the invariant cross-section for the production of the secondary particles (referred to as the Lorentz transformation) is assumed to be expressed as a function of both \( X (= \frac{E}{E_0}) \) and \( P_t \) and is then given by

\[
E \frac{d^3\sigma}{dP_t^3} = f(X, P_t), \tag{7.5.6}
\]

where \( X \) is the energy of the secondary particle expressed in units of the energy of the primary particle, and named the Feynman variable. The ideas similar to this scaling law have been known since the early days in the research on cosmic rays.

The transverse momentum \( P_t \) remains constant in Eq. (7.5.6) with respect to the Lorentz transformation in the colliding direction, and also takes a nearly constant value of \( 300 \sim 400 \text{ MeV}/c \), being independent of \( E_0 \), as shown experimentally in Fig. 7.5.3.\(^5\) It is, therefore, possible to separate Eq. (7.5.6) as

\[
f(X, P_t) = F(X)F'(P_t). \tag{7.5.7}
\]

This equation is rewritten as
\[
\frac{d\sigma}{dP_t} = \frac{dP_t}{E} F'(P_t) \frac{dP_\perp}{P_\perp} \quad (7.5.8)
\]

Using the definition \(P_t^2 + m^2 = m_\pi^2\), though \(E^2 = P^2 + P_t^2 + m^2\) and taking into account the approximation \(E \approx P_\perp\) in the high energy limit of \(m \ll E_0\), Eq. (7.5.8) is further rewritten as

\[
\frac{d\sigma}{dP_t} = F(X) \frac{dX}{X} F'(P_t) \frac{dP_\perp}{P_\perp} \quad (7.5.9)
\]

The two terms on the right-hand side are, respectively, given as

\[
\frac{F(X)}{X} = (1 - X)^m X^{-1} \quad \text{and} \quad m = 3 \sim 4
\]

and then

\[
F'(P_t) = e^{-c P_t} \quad \text{and} \quad c \approx 6.
\]

The multiplicity of the secondary particles is expressed as

\[
n = \frac{1}{\sigma_0} \int d\sigma = \frac{2}{\sigma_0} \int e^{-c P_t} dP_t \int_{X_{\min}}^{1} F(x) \frac{dX}{X} \\
\approx AF(X_{\min}) \ln X_{\min}. \quad (7.5.10)
\]
Since $X_{\text{min}}$ is given by $m/E_0$ from the earlier definition, the result $n \propto \ln E_0$ is obtained by substituting this into the above equation. In so doing, we assumed that $F(m/E_0) \approx 1$ and neglected the integration constant, which is negligibly small. It may be said that the above result obtained theoretically agrees rather well with the observed results in the low energy region as shown in Fig. 7.5.4, whereas the theoretical result is too small to explain the observed results in the high energy range. This one-dimensional scaling law as obtained above only shows that, as regards the distribution of the secondary particles on the $Y$ axis, the axial momentum $P_0$ becomes larger with $E_0$ on the $Y$ axis as indicated by the dotted lines in Fig. 7.5.5. Thus, it follows that the integrated result is only proportional to $\ln E_0$. Since the production rate of the secondary particles as measured in units of $Y$, however, has a tendency to increase in practice, this rate becomes as shown by black lines in this figure. In order to include this tendency, the radial scaling law has been devised. According to this law, the multiplicity of the secondary particles is expressed as

$$n \propto \ln E_0 + b(\ln E_0)^2. \quad (7.5.11)$$

It should be noted here that the result shown in Fig. 7.5.4 is also well approximated by the form $e^{-bX}$.

**BRAZIL-JAPAN COLLAB.**
- $40 > \Sigma E_Y \geq 15 (24)$
- $15 > \Sigma E_Y \geq 10 (40)$
- $10 > \Sigma E_Y \geq 7 (49)$

TeV (EVENTS)

<table>
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<tr>
<th>Number of $\gamma$-rays per event</th>
<th>$X' = E_Y/\Sigma E_Y$</th>
</tr>
</thead>
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<td>0.7</td>
</tr>
<tr>
<td>0.6</td>
<td>0.6</td>
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<tr>
<td>0.5</td>
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<tr>
<td>0.4</td>
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<td>0.1</td>
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<tr>
<td>1</td>
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</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Fig. 7.5.4. X-distribution.
7.5.3 Propagation in the atmosphere

After entering the atmosphere, all of the components in the primary cosmic rays are destroyed into nucleonic and meson components as a result of their collisions with the atmospheric nuclei. The pattern which these secondary components produce deep into the atmosphere can be investigated by solving the diffusion equations for these components in the vertical direction. As regards the nucleonic components, this equation is expressed as

\[
\frac{dN(E, t)}{dt} = - \frac{N(E, t)}{\lambda_N(E)} + \int_0^\infty \frac{N(E', t)F_{NN}(E, E')}{\lambda_N(E')} \cdot \frac{dE'}{E},
\]

(7.5.12)

where \(F_{NN}(E, E')\) gives the probability that the nucleons of energy \(E'\) produce those of energy \(E\). The diffusion equation for pions (\(\pi\) mesons) which are the main component of the secondary particles is written as

\[
\frac{d\pi(E, t)}{dt} = - \frac{(E, t)}{\lambda_\pi(E)} - \frac{B_\pi \pi(E, t)}{Et} + \int_0^\infty \frac{\pi(E', t)F_{NN}(E, E')}{\lambda_\pi(E)} \cdot \frac{dE'}{E} + \int_0^\infty \frac{N(E, t)F_{NN}(E, E')}{\lambda_N(E)} \cdot \frac{dE'}{E}.
\]

(7.5.13)

When we assume, in this equation, that the mean atomic weight of the atmosphere is 14.5, the mean-free path is given by

\[
\lambda_\pi(\text{g/cm}^2) = 2.4 \times 10^4 / \sigma(\text{mb}),
\]

(7.5.14)

where \(\sigma\) is a function of \(E\). The second term on the right-hand side of Eq. (7.5.13) is related to the decay of pions and is given, as well known, by

\[
B_\pi = \frac{Hm_\pi}{\tau_\pi} \approx 120 \text{ GeV},
\]

(7.5.15)

where \(H=\rho=6.4\text{km}\), and \(\tau_\pi\) and \(m_\pi\) are the life time and the mass of a pion, respectively. The solutions for Eqs. (7.5.12) and (7.5.13) are obtained as\(^{15}\)
\[ N(E, t) = N(E, 0)e^{-t/\lambda_a} \]  

(7.5.16)

and

\[ \pi(E, t) = N(E, 0)Z_{N \pi} t \frac{E}{\lambda_N} e^{-t/\lambda_a} \frac{E}{E + B_\pi}, \]  

(7.5.17)

where

\[ \lambda_a = \lambda_a/(1 - Z_{ab}) \]  

(7.5.18)

and

\[ Z_{ab} = \int_0^1 X^{-1} F_{ab}(X) \, dX. \]  

(7.5.19)

In the energy region where \( E \gg B_\pi \), the contribution of pions becomes larger with atmospheric depth. In these above equations, the suffices a and b denote either \( N \) or \( \pi \). If the energy spectrum is expressed as \( E^{-1.7} \), for the primary cosmic rays in applying this scaling law, the fragmentations to produce the secondary particles in the upper part of the atmosphere are most effective in the calculation of \( Z_{ab} \). The energy spectrum for the secondary particles thus always becomes \( E^{-1.7} \).

Figure 7.5.6 shows the results where these calculations have been compared with observations. They are both well coincident with each other in the upper portion of the atmosphere, but are farther away from each other deep in the atmosphere. The energy spectrum deduced from the observations declines more sharply compared with that which has been obtained from the scaling law. It seems, however, that this discrepancy between the observed and the theoretical results can be eliminated by taking into account three properties described in the following.

(a) The power index of the energy spectrum for the primary cosmic rays has a tendency to become larger with particle energy. The nucleon components which repeat collisions \( m \) times in the atmosphere should have a larger power index in order for them to correspond to the energy higher by \((1 - K)^{-m}\), where \( K \) is the inelasticity.

(b) In the approximation of \( n \sim E\alpha \), the multiplicity tends to increase with \( \alpha \) being given as \( \alpha \approx 0.1 \sim 0.2 \), because their interactions are a little different from those expected from the scaling law.

(c) Since the power index \( \delta \) is given as \( 0.01 \sim 0.02 \), where the interaction cross-section is expressed as being proportional to \( E\alpha \), the energy spectrum necessarily changes by the order of \( m\delta \) after \( m \) collisions (Fig. 7.5.7).

7.5.4 Characteristics of other interactions

a) Large transverse momentum \( P_t \)

As has already been explained, the transverse momentum \( P_t \) follows the distribution law of the form \( e^{-6P_t} \). The mean value of \( P_t \) is, therefore, calculated as about 300 MeV/c by integrating this law multiplied with \( P_t \) with respect to \( P_t \). This mean value is
Fig. 7.5.6. The observed results at the top of Mt. Fuji on the γ-ray intensity resulting from the π⁰→2γ in the atmosphere for (a) the atmospheric depth and (b) the energy spectrum.
almost independent of \(E_0\) as shown earlier. However, there exists a large component of \(P_t\), which is proportional to \(P_t^{-4}\) in the fringe region for momenta higher than \(\sim 2\) GeV/c in the above distribution. The existence of such a component suggests that the hard collisions\(^{16}\) take place as depending on the quark-parton models based on the fundamental particles in nucleons (Fig. 7.5.8). It is here noted that the existence of large \(P_t\), which tends to increase with \(E_0\), is not rare in the energy range common in cosmic ray phenomena. The observed results are often presented using the integral form, but the distribution law experimentally obtained on this form is given as that which is proportional to \(P_t^{-2}\).

b) **Centaurus**\(^{16}\)

In the Japan-Brazil cooperative work conducted on mount Chakaltaya (5200 m), an interesting phenomenon, named “Centaurus”, was discovered which resulted from the nuclear interactions in the energy region close to \(10^{15}\) eV in the atmosphere. The feature of this phenomenon is explained as follows: in this phenomenon, nucleons and anti-nucleons seemed to consist mostly of several tens of secondary particles and, strangely, there were few pions contained in this event. This ultra-high-energy phenomenon may contain some interactions substantially different from those seen in relatively lower energy phenomena, since it seems difficult to understand this phenomenon as just an extension of the latter to the ultra-high-energy region.

The observational results on these ultra-high-energy phenomena are now being obtained from the measurements by emulsion chambers on board balloons and airplanes, or located at high mountain ranges. The emulsion chambers consist of nuclear emulsions and some matter slabs, both of which are alternately piled up, and are especially useful for exact measurements of the energies of the secondary particles.
produced during the development of the electromagnetic cascade showers initiated by gamma rays or electrons (see Figs. 7.5.9 and 7.5.10). When the shower production layer is inserted between two chambers divided as shown in Fig. 7.5.10, it becomes possible to distinguish the incident particles from the secondary particles during the development of these showers.

7.6 Extensive Air Showers

After entering the top of the atmosphere, the primary cosmic rays of ultra high energies begin to collide with the atmospheric nuclei and produce an enormous number of secondary particles. While repeating successively their collisions with the atmospheric nuclei along their paths, most of these particles produce the nucleonic cascade showers along the incident direction of the primary cosmic rays. The electromagnetic cascade showers are developed around the axis of the nucleonic cascade showers after being initiated by high-energy gamma quanta produced from the pion decay \( \pi^0 \rightarrow 2\gamma \). Furthermore, a number of muons, due to their decay \( \pi^+ \rightarrow \mu^+ + \nu_\mu \), come down simultaneously with the components of those two showers. Such phenomena are defined as extensive air showers, or, simply, the air showers.

These phenomena are usually observed by distributing various particle detectors over a wide area, on the ground or on the top of a mountain, which is extended from several hundred to a thousand \( m^2 \). Using these detectors, much information is obtained on the locations of the cores of the air showers, their directions, the
distribution function and the number density for each component constituting the showers. In recent times, the Cerenkov light emissions in the atmosphere and the atmospheric scintillations have been observed to search for the developing pattern of the air showers in the atmosphere, but their observations are limited to the clear nights with no moon light.

The information mentioned above gives us some clues to understand the energy spectrum and the chemical composition of the primary cosmic rays and the nature of the ultra-high-energy nuclear interactions. The research on extensive air showers is, therefore, based on the analyses of the observed data indirectly obtained on them, but it may be emphasized that this is the only means for the research on nuclear interactions in the energy region higher than $10^{16}$ eV.
7.6.1 Electron component

Since about 90% of the charged particles in an air shower are identified as the electron component, it is essential to measure the numbers and the characteristics of these electrons. Although a number of electromagnetic cascade showers initiated from different nucleon cascade showers make a composite structure in the air shower, the method widely applied now is that which analyzes this composite structure viewed as an average feature of the electromagnetic cascade shower.

As has been described in Chapter 3, an electromagnetic cascade shower first develops while repeating the processes of the creations of electron-positron pairs from gamma rays and the bremsstrahlungs from both electrons and positrons in the atmosphere. Then, the numbers of these particles rapidly increase by these processes and eventually reach a maximum. After the mean energy of these particles reaches some critical energy, the shower gradually begins to damp due to the action of ionization losses and the Compton effect. In the atmosphere, the radiation path length $X_0$ and the critical energy $e_0$ are given by 37 g/cm$^2$ and 84 MeV, respectively. The Moliere unit $r_0$ to show the lateral extension of the air shower is expressed as

$$r_0 = \left( \frac{21}{e_0} \right) X_0 \text{ g/cm}^2.$$  \hspace{1cm} (7.6.1)

This unit is about 80 m at ground level and about 100 m at the top of a mountain 3000 m high.

The lateral distribution of the electron density is determined by both the distribution of the directions for the decay $\pi^0 \rightarrow 2\gamma$ and the transverse extension by the Coulomb scattering effects in the developing stage of the electromagnetic cascade shower, and is approximately expressed by the N-K-G (Nishimura-Kamata-Greisen) function$^{17}$ as

$$\Delta(r, s) = Ac(s)r^{s-2}(1 + r)^{-s-4},$$  \hspace{1cm} (7.6.2)

where $s$ and $c(s)$ are the age parameter and the normalization factor, respectively. The distance $r$ is in Moliere units. Comparing this function with the experimental results, the age parameter $s$ can be obtained for each air shower. The integration

$$N_e = \int_0^\infty \Delta(r, s)2\pi rdr$$  \hspace{1cm} (7.6.3)

gives the total number of electrons (Fig. 7.6.1). Although the parameter $s$ does not necessarily correspond to the age of a single air shower, it gives an idea close to the age of the shower. In fact, the examples in which $s$ is young show that the mean energy of the electron component is relatively higher and also the damping constant is larger (Fig. 7.6.2).

Let us consider now the transition curve of the air shower. Figure 7.6.3 shows the measured results on the relation of atmospheric depth with the size of the air shower, $N_e$, whose frequency is constant at different depths in the atmosphere. Taking into
Fig. 7.6.1. The electron density distribution as given by parameter $s$.

Fig. 7.6.2. The relation of the air shower size with the mean shower age $\bar{\tau}$.
account that the air showers initiated by the primary cosmic rays of large $E_0$ statistically correspond to those which produce large $N_e$, this figure indicates the mean transition curve of the air showers in the atmosphere. Since the atmospheric depth $t_{\text{max}}$ for $N_e$ to reach a maximum is written as

$$t_{\text{max}} = a + b \log E_0$$  \hspace{1cm} (7.6.4)

and also $N_{\text{em}} \propto E_0$, the log $N_e$-$t$ diagrams are similar to each other as shown in Fig. 7.6.3. The value of $b$ is about 90 g/cm$^2$ for electromagnetic cascade showers, but about 50 g/cm$^2$ for air showers. Thus, the development of the latter is faster than that of the former.

The size $N_e$ of the air shower may give the best estimate of the total energy of the shower, namely, the energy of the incident particle $E_0$. If we write

$$E_0 = R(t)N_e(t),$$ \hspace{1cm} (7.6.5)

the magnitude of $R$ is $(1 \sim 1.5) \times 10^9$ eV near the region of $t_{\text{max}}$ and about $10^{10}$ eV at ground level (where $R$ is about equal to 1000 g/cm$^2$).
7.6.2 Nucleonic components

The air shower in the early stage of development only consists of the nucleon cascade showers which develop along the central axis of the shower. When the air shower enters the damping stage after \( N_e \) has reached a maximum, where this shower is fully developed, the nucleonic components no longer largely influence the development of the shower. Almost all of the air showers observed at the top of mountains and at ground level are in the damping stage and consequently do not contain many high-energy nucleons.

Since \( P_t \) of the nucleonic components is low when they are produced in the atmosphere and they are rapidly damped, they are found only near the central axis of the shower and their transverse distribution is approximated by \( e^{-r/r_0} \). \( r_0 \) is dependent on the primary energy and is given by

\[
\begin{align*}
    r_0 &\sim 1 \text{ m for } > 1 \text{ TeV} \\
    r_0 &\sim 1.5 \text{ m for } \sim 100 \text{ GeV} \\
    r_0 &\sim 7 \text{ m for } \sim 10 \text{ GeV}
\end{align*}
\]

and

\[
r_0 \sim 20 \text{ m for } \sim 400 \text{ MeV}.
\]

By integrating this transverse distribution with respect to energy, the energy spectrum of the nucleonic components is obtained as follows: this spectrum is given as \( AE^{-1.4} \) in the energy range less than 1 TeV, but becomes \( A'E^{-2-2.5} \) in the energy range higher than 1 TeV. The intensity of these components of energy higher than 100 GeV is of the order between 10 and 20 for an air shower of magnitude \( 10^6 \). The relation of the nucleon number \( N_N \) with the size \( N_e \) is given as

\[
N_N \propto N_e^{0.75}.
\]

(7.6.6)

7.6.3 Muon component

The muons in the air shower are produced from the decays of pions and K-mesons near the maximum of the nucleon cascades—7 to 10 km above the ground—and come down through the atmosphere whilst widening their distribution in space, though they move almost straight along the direction of the incident primary particle. Their density distribution at ground level is given by

\[
\Delta \mu(N_e, r) = 18 \left( \frac{N_e}{10^6} \right)^{0.75} r^{-0.75} \left( 1 + \frac{r}{320} \right)^{-2.5}.
\]

(7.6.7)

Therefore, the total number of muons is clearly proportional to \( N_e^{0.75} \) and their distribution shows a rapid decrease of muons with distance from the central axis of the shower (see Fig. 7.6.4). While the muons of higher energy are concentrated near
the central axis, the energy spectrum as a whole shows the integral form expressed by $E^{-1.4}$ like the nucleonic components in the last section. Although only a few muons of energy higher than 100 GeV have been observed, it is inferred that their energy spectrum follows the form $E^{-2.5}$ since they are also decay products.

7.6.4 Problems related to the extensive air showers

As the accuracy on the observations of the air showers becomes higher, it seems that the properties of the primary cosmic rays and the characteristics of the ultra-high-energy interactions will be more clearly understood. However, there may exist some problems to be studied in the near future.

a) Central cores

In general, both electrons and high-energy nucleons are usually concentrated in the area around the central axis of the air shower, called the core, but the distribution
of the electron component, i.e., that of the energy flow, in the area within 2~3 m from the central axis, is different from one shower to the other.

On occasion, this distribution shows the double core structure and, though it is rare, a multi-core structure is sometimes observed in the development of the air shower. The phenomena like these are now understood to show the characteristics of the nuclear interactions during the initial phase of the development of the air shower. Furthermore, it is thought that the transverse momentum, $P_t$, generally reaches the order of several GeV/c in the ultra-high-energy region.

b) Fluctuation in the air showers

Even for the air showers initiated by primary particles of the same $E_0$, the patterns of their later developments seem to be much different from each other, since the development of the air shower is highly dependent on physical quantities such as the chemical composition and the energy distribution of the primary cosmic rays and the rate of the energy transfer to the electromagnetic components in the early phase of their development. However, the important problem still remains on the heights where the first collisions of the primary particles with the atmospheric nuclei take place, since these collisions almost determine the fluctuations in the air showers. Figure 7.6.5 shows an example related to this problem, which explains that the size of the air shower initiated by a primary particle of low $E_0$ after entering deep into the atmosphere is observed as that which is initiated by a primary particle of high $E_0$ near the top of the atmosphere. The effect of this problem on the analysis of the shower phenomena becomes very large with the sharpness of the energy spectrum for the primary cosmic rays.

The problem on the damping constant in the air shower will be considered here as an example. If we define the damping path length measured using the zenith angle distribution, the size spectrum and the transition curve of the air shower by $\lambda_{ob}$,
\[ I(>N_e) = AN_e^{-\gamma} \quad \text{and} \quad N_e(t) = N_0 e^{-t/\lambda}, \quad (7.6.8) \]

we obtain for \( \lambda_{ob} \)

\[ -\frac{1}{\lambda_{ob}} = \frac{\partial \ln I}{\partial t} = \frac{\partial \ln N_e^{-\gamma}}{\partial t} = \frac{\partial \ln e^{-t/\lambda}}{\partial t} = -\frac{\gamma}{\lambda}. \quad (7.6.9) \]

Hence,

\[ \gamma \lambda_{ob} = \lambda. \quad (7.6.10) \]

Since \( \lambda_{ob} \approx 110 \text{ g/cm}^2 \) for a wide range of \( N_e \) and \( \gamma \approx 2 \), it follows that \( \lambda \approx 220 \text{ g/cm}^2 \).

On the other hand, \( \lambda \) is estimated from the rate of energy loss as

\[ \lambda = \frac{\sum E}{\Sigma (dE/dt)} = \frac{N_e \langle E_e \rangle}{N_e (dE/dt)} \approx 120 \text{ g/cm}^2, \quad (7.6.11) \]

where the mean energy \( \langle E_e \rangle \) corresponds to the energy for an electron, though including gamma-ray and other energies.

Although the results derived from Eq. (7.6.9) and Eq. (7.6.10) do not agree with each other, the derivation of Eq. (7.6.9) does not seem to be correct according to the calculations by Miyake.\(^{19}\) For the case where the fluctuations are large, \( \lambda_{ob} \) is written as

\[ \lambda_{ob} = L(1 + r), \quad (7.6.12) \]

where \( L \) is the collision mean free path given by \( \approx 80 \text{ g/cm}^2 \), and the mixing ratio, \( r \) of the air showers initiated by the primary cosmic rays of low \( E_0 \) is given by \( \approx 0.5 \). Using these numerical results, the observed values of the damping path length \( \lambda_{ob} \) are well interpreted. This suggests that the divided transfer of the primary energy \( E_0 \) to the energies of the secondary particles occurs very roughly in the high-energy interactions, and the tendency as seen on the transition curves does not change much even if \( N_e \) is large.

7.7 **Muons and Neutrinos**

As the result of the collisions of the primary cosmic rays with the atmospheric nuclei, many pions and K-mesons are multiply produced. Because the life times of these mesons are extremely short (\( \approx 2 \times 10^{-8} \) sec), muons and neutrinos are further produced from some of them due to their decays in flight. The life time of muons is relatively longer (\( \approx 2 \times 10^{-6} \) sec) and is further extended by the Lorentz factor in the high energy region (see Chapter 3). In consequence, the moving muons are able to survive for a time much longer than their proper life at rest and to penetrate deep underground, since they never make strong interactions with other particles. In fact,
the lengthening of time as predicted by the theory of special relativity was first established from the observations of cosmic ray muons. Neutrinos are stable and have no charge. Since their interactions with matter are extremely weak, they are able to penetrate almost freely through the whole body of the earth.

There are three methods to measure the momentum spectra of muons. The first uses the momentum spectrometer equipped with a range detector to which a magnetic field is applied. The second method measures the relation of the depth underground with muon intensity. The energy spectrum of these muons is then calculated from the measured results. The last one is to obtain the size spectra for the bursts from the measurements of the bremsstrahlungs by muons using the emulsion chamber or the calorimeter.

Next, the method to calculate the muon intensity will be considered. After multiplying $\pi(E, t)$ given in Eq. (7.5.17) with the decay probability of muons between $t$ and $(t+\,dt)$, the equation

$$
d\pi = - \frac{B_\pi(E, t)}{E_\pi} \, dt \tag{7.7.1}
$$

is integrated with respect to $t$. In deriving the result after integration, if we define the relation

$$
E_\mu = \frac{m_\mu}{m_\pi} \frac{E}{E_\pi} = r E_\pi, \tag{7.7.2}
$$

the intensity of muons at the depth $t_0$ for the upper limit of the integration is obtained as follows: if we assume that $t_0$ is large enough, this intensity is given by

$$
\mu(E, t_0) = Z N_e N \left( \frac{E}{r}, 0 \right) \frac{\lambda_\pi}{\lambda_N} \cdot \frac{r B_\pi}{E + r B_\pi}. \tag{7.7.3}
$$

Since the decay and the ionization loss of muons in the atmosphere are both neglected in the derivation of the above formula, this formula can only be applied to the high energy region.

### 7.7.1 Momentum spectrometer

Iron-core electromagnets are often used to produce the magnetic fields for the measurements of muons. The energy loss of muons due to their passage through an iron slab is negligible in the high energy region, and a magnetic field of intensity as high as 17 K gauss can be easily generated by such magnets, since their magnetic resistance is very low. The Coulomb scattering of muons in the iron slab is given by

$$
\Delta \theta = \frac{21(\text{MeV})}{E} \sqrt{t}, \tag{7.7.4}
$$
where $t$ is measured with the radiation path length in iron. Since the deflection angle in the magnetic field is given by $300 \times H \rho = E$, where $H$, $\rho$ and $E$ are, respectively, the magnetic field (gauss), the curvature radius in cm and the muon energy in eV, the angle $\theta$ is given by

$$\theta = \frac{l}{\rho} = \frac{300Hl}{E}. \quad (7.7.5)$$

Hence the magnitude of $\Delta \theta / \theta$ is constant and independent of $E$, where $l$ is the scale length of the magnetic field. As a result, it is easy to make this magnitude less than 20% by making the thickness of the iron slab thick enough. The electromagnets of large size named DAIS and MUTRON are being now in use in West Germany and Japan, respectively. The highest detectable momenta of muons by DAIS and MUTRON are 7 TeV/c and 2 TeV/c, respectively.

Figure 7.7.1 shows the energy spectra for muons at zenith angles $0^\circ$ and $90^\circ$. The muon intensity is relatively low for large zenith angle because of both the ionization loss and the natural decay of muons in the low energy region, while this intensity becomes higher in the high energy region since the production probability of pions is large for their penetration into the atmosphere with large zenith angle. The dependence of this intensity on zenith angle is well understood by the transformation of the term $rB\pi/(E+rB\pi)$ in Eq. (7.7.3) to $rB\pi \sec \theta^*/(E+rB\pi \sec \theta^*)$, where $\theta^*$ denotes the zenith angle close to the top of the atmosphere. Thus, the case for $\theta=90^\circ$ is
equivalent to the case for $\theta^* = 85^\circ$, for instance.

The momenta and the charge of muons are simultaneously determined by the measurement with the electromagnet. The cause for the charge ratio defined as $\mu^+ / \mu^-$ always being larger than unity is due to the fact that many more protons than antiprotons are contained in primary cosmic rays (Fig. 7.7.2). The range of this ratio is generally between 1.25 and 1.35, but has a tendency to become even larger since the number of muons increases due to the decays of pions and K-mesons. In the energy range less than 100 GeV, the decays of pions play an important part to raise this ratio, while K-meson decay becomes relatively more important in the energy range higher than 500 GeV.

7.7.2 Underground observations

Hadronic and electromagnetic components are both rapidly absorbed under the ground and eventually only muons and neutrinos remain there. The depth under the ground is generally represented by $hg/cm^2$ or in the unit of m.w.e. (abridged form of meter water equivalent). Figure 7.7.3 shows the relation of this depth with the muon intensity, which has been measured at the Kolar gold mine in the south of India as part of the Japan-India cooperative research. This relation is experimentally expressed as

$$I_\perp(h) = \frac{A}{h + H} (h + a)^{-\alpha} e^{-\beta h} \text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1},$$  

(7.7.6)

where $A=174$, $H=400$, $a=11$, $\alpha=1.53$ and $\beta=8 \times 10^{-4}$. In this equation, the three terms on the right-hand side express three physical quantities as shown in the

![Graph](image)

Fig. 7.7.2. The charge ratio of cosmic ray muons (the number ratio of $\mu^+$ to $\mu^-$). ●: The measurements by the University of Durham (ground level, $\theta=0^\circ$). ▲: the measurements by Kiel University (ground level, $\theta=0^\circ$). △: the measurements by Kiel University (ground level, $\theta=75^\circ$) and ◼: the measurements by the University of Utah (underground, various azimuth angles).
The relation between the atmospheric depth and the intensity of muons at the Kolar Gold Field in India.

Following: the first and the second terms respectively give the decays of mother particles and the muon initial spectra, while the third term approximates both the energy and the depth of muons at a great depth in the atmosphere. The energy loss of muons is given\(^{11}\) by

\[
\frac{dE}{dh} = 184 + 7.6 \ln \frac{E}{m_0 c^2} + bE
\]

where \(a = 25 \text{ MeV/hg-cm}^2\) and \(b = (4 \sim 5) \times 10^{-4} / \text{hg-cm}^2\). The factor \(a\) denotes the ionization loss and the loss by knock-on electrons, while \(b\) gives the loss index due to bremsstrahlung, electron-positron pair creation and nuclear interactions. By integrating the above equation with respect to \(E\), we obtain
\[ E = \frac{a}{b} \left( e^{bh} - 1 \right). \] (7.7.8)

This equation gives the relation between the energy and the range underground of muons. Although Eq. (7.7.6) can be transformed to the energy spectrum by referring to the relation given by Eq. (7.7.8), a large variation in the bremsstrahlung loss should be considered in doing so, because Eq. (7.7.8) only gives the average energy loss. In fact, the result shown in Fig. 7.7.4 indicates that the distribution of the ranges is wide due to such variability. The energy spectrum for muons by taking into account the above correction is thus given by, in GeV units,

\[ I(\geq E) = \frac{3}{E + 80} \left( E + 2 \right)^{-1.55} \text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}. \] (7.7.9)

This spectrum in the energy range higher than 1 TeV (in TeV units) is further expressed as

\[ I(\geq E) = 5.0 \times 10^{-8} E^{-2.6} \text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}. \] (7.7.10)

As to the underground experiments, various research such as on the nuclear interactions with virtual photons associated with muons, on muons stopped in absorbers with constant thickness and on muons in parallel flight in the same time has seen in progress.

The observations of neutrinos, on the other hand, are done deep underground in order to reduce the background radiations, since the cross-section for neutrino
interactions is extremely small. Since the total cross-section for the interactions of neutrinos with nucleons is small, given by

$$f_{\text{FN}} = 0.74 \times 10^{-38} \text{ } E_f \text{ cm}^2,$$

(7.7.11)

the earth is almost perfectly transparent to neutrinos. Since muon-neutrinos are produced together with muons by pairs, the energy spectrum for them can be automatically obtained only by transforming the energy of associated muons to that of neutrinos.\textsuperscript{22)} Figure 7.7.5 shows the relations of the pairs of muons with muon-neutrinos at the time of the decays of pions and K-mesons. Figure 7.7.6 indicates that the conditions for muon-neutrinos to be generated are nearly equal to any two of muons if the zenith angles of their incidence to the earth are the same as each other. The outline of the neutrino detector system built in the Kolar gold mine during the years from 1965 to 1970 is shown in Fig. 7.7.7. Seven detectors are arranged horizontally and their sizes range from 4 to 8 m\textsuperscript{2}. Since the muons produced in the atmosphere are mostly concentrated in a small angular area around the zenith, all of

![Diagram of neutrino interactions and paths](image_url)

Fig. 7.7.5. The relation between muons and muon-neutrinos in the K-meson decays. (a) $\pi$-mesons (pions), (b) Kaons.

![Diagram of neutrino detector system](image_url)

Fig. 7.7.6. Paths of muon-neutrinos.
Fig. 7.7.7. Neutrino detectors at the Kolar Gold Field station.

Fig. 7.7.8. The energy spectrum of muons.
the muons arriving from the directions with large zenith angle are generated from the nuclear interaction with neutrinos. The intensity of these muons is expressed as

$$I_{\nu} = \iiint A\Omega(\theta, \varphi)I_{\nu}(E_\nu)\sigma(E_\nu)R(E_\nu)dE_\nu d\Omega.$$ \hspace{1cm} (7.7.12)

This intensity is experimentally given as $I_{\nu} = 3 \times 10^{-13}$ cm$^{-2}$ sec$^{-1}$ sr$^{-1}$.

Figure 7.7.8 shows the energy spectrum for muons. Using this spectrum, it is possible to calculate the energy spectrum for the primary cosmic rays. The result shown in Fig. 7.7.9 indicates the comparison of this spectrum with the energy spectrum for the primary cosmic rays deduced by Ryan and Grigorov from the observations. The discrepancy between these two spectra can be explained by taking into account the $E_0$-dependence of the multiplicity as a whole on the production of muons.

According to the first approximation, muons are mainly produced at a depth about 100 g/cm$^2$ from the top of the atmosphere. Thus, the gradient of the energy spectrum for muons becomes similar to that for the primary cosmic rays except for the term for the muon decays. It is also estimated that the gradient of the energy spectrum for neutrinos becomes sharper due to the difference of the energy for muons.

Fig. 7.7.9. The energy spectrum of the primary cosmic rays as deduced from the muon spectrum.

REFERENCES