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Short Contribution

Upwelling by a Wave Pump

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A rigid open-ended pipe is submerged in the ocean below the troughs of the surface waves and held fixed in the vertical position, the lower end being at or below the depth of wave influence. When surface gravity waves propagate past the pipe, water flows up as long as waves are present. The steady upward vertical velocity in the center of the pipe is calculated to be proportional to the square of both the average wave steepness and the pipe's radius. An application is to bring nutrient rich waters up into the sunlit surface layers of the open oceans.

Keywords:
· Wave pump,
· upwelling.

1. Introduction

Within the past 50 years two different ideas have been proposed for water pumps applicable to the ocean and both of them have been built and proven to work under environmental conditions. First, in historical order of "invention", there is the *perpetual salt fountain* concept that is based on the fact that both temperature and salinity affect the density of seawater and the constraint that heat can be conducted through a metal pipe whereas salt cannot. This idea attracted only a brief interest in the 1950s (Stommel *et al*., 1956) but its origin probably goes back about 100 years before that to meteorology (Veronis, 1981). In a region of the ocean where warm salty water overlies colder fresher water (e.g. some areas of the tropics and subtropics), such that overall static stability in the vertical direction is maintained, a metal pipe is placed in a vertical position so that it bridges these two water types. The pump is primed by moving some water upward inside the pipe. Heat will diffuse into the rising water in the pipe through its conducting metal walls but the impermeability of the pipe will prevent any salt from diffusing in. Thus the water in the pipe will become less dense than the environment at the same depth and will continue to rise upward under positive buoyancy. This is the perpetual salt fountain, which was first tried in the North Atlantic in 1971 using a 1000 m flexible plastic tube, but the results were unconvincing (Huppert and Turner, 1981). However recent results in 2002 in the North Pacific have been more encouraging (Maruyama *et al*., 2004).

The salt fountain is also reversible because if the water inside the pipe were initially given a downward push, water would continue to flow downward by the analogous effect of heat but no salt diffusing out of the sinking water in the pipe, causing a negative buoyancy. Since diffusion is what drives the fountain, the time-scale of this type of pump is a diffusion time-scale, which is expected to be relatively long, compared to a wave period, for example, while the flow velocity is anticipated to be relatively weak. In a different sense the salt fountain is not reversible because it cannot be made to work if a layer of cold fresh water overlies a layer of warmer saltier water, as occurs above the Mediterranean outflow and in some high latitude regions (e.g. Norwegian fjords).

It was subsequently pointed out that for the mechanism to work the pipe is superfluous (Stern, 1960), because in water heat diffuses about 100 times faster than salt, and so *salt fingers* were born on paper and shortly afterward exhibited in the laboratory.

Second, there is the idea for a *wave powered pump* initiated by Isaacs during the period 1974–1976 which involves a buoy moving up and down on the wave surface, a pipe attached to the buoy and a flapper valve that opens and closes inside the pipe (Behrman, 1992). The way it is hinged allows the valve to open and shut at opposite phases of the wave cycle, causing water to rise upward. If the valve is hinged in the opposite way, the pump can be made to move water downward. The time-scale for this pump is the wave period, which is relatively short in comparison to the diffusion time-scale. A major motivation behind the concept of the wave-powered pump was to extract energy from the surface gravity waves in the ocean.

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A third apparently novel type of wave water pump proposed below is mechanically simpler than the Isaacs pump, because it involves no valves at all and has a considerably faster pumping rate than the salt fountain method. In addition, this wave pump does not need to be primed to get it started unlike the salt fountain. The basic idea is derived from the experience of submarines at sea. A great feature of submarines is that they can ride out storms comfortably by remaining at or below the depth of wave influence. However, should there be a need to surface, it can be dangerous. When gravity waves are of sufficiently great height and length and the submarine enters the depth of wave influence, there is a pressure difference across the diameter of the submarine in the vertical direction due to the larger orbital particle speed of the waves on the top compared to that on the bottom of the submarine. Consequently, the net Bernoulli effect means that the rate of ascent cannot be controlled and the submarine will rise to the surface too quickly. Imagine the submarine to be a rather large pipe oriented in the horizontal position. Then instead of having the pipe itself move upward in the wave zone, why could this same Bernoulli effect not be used to pump water upward through a much smaller pipe that is held fixed in the vertical position?

Suppose a strictly rigid, impermeable pipe is oriented vertically and submerged in the ocean near the surface. It is held in a fixed position so that it will not oscillate up and down with the imposed wave motion. Both ends of the pipe are open and the constant diameter of the pipe is taken to be much smaller than the wavelength of the surface gravity waves that pass by it, so as to minimize any disturbance to the wave motion by the pipe and to insure that the only forces inside the pipe must be directed parallel to the pipe. (There is no requirement that the pipe's diameter be infinitesimally small.) Its lower end is located at or below the depth of wave influence, where the fluid motion and pressure variations due to the surface wave are negligibly small compared to those at the surface. The upper end of the pipe is placed just below the lowest anticipated level of the wave troughs in the region of the most active part of the wave motion. At most 15 m below mean sea level should be sufficient for the depth of the upper end of the pipe because, although a 30 m wave height is possible, it is a very rare occurrence. (For example, a slightly negatively buoyant pipe could be suspended by a line from a float on the surface and if the pipe has sufficient inertia, it will not move while the float rides up and down with the waves. The pipe could also be attached to a piling of a pier, oil platform, FLIP, etc.)

Once in position, water will immediately begin to flow upward inside the pipe and it will continue to do so as long as surface waves move by the pipe. There is no need to prime the pump because as soon as a pressure gradient is established along the length of the pipe, fluid will move upward. (The nature of a fluid is to respond to the least amount of force.) Bernoulli's principle is the key to running this wave pump, but the theoretical exponential decay of the wave motion with increasing depth in deep water is also essential. Particle motion due to the wave moves across the top opening of the pipe, causing relatively low pressure there compared to the essentially constant and higher pressure at the bottom end of the pipe. The pressure difference along the length of the pipe causes the fluid to flow consistently upward and never downward, like the old-fashioned stomach pump, fluid atomizer or insect sprayer.

A possible point of confusion is the fact that in the absence of a pipe there is no mean vertical velocity associated with a surface gravity wave of constant amplitude, according to linear or nonlinear theories. (A mean horizontal velocity exists, which is the second order Stokes drift velocity.) Why does putting a pipe into the ocean change this picture? Inside a pipe of diameter much smaller than a wavelength the forces associated with the wave motion cannot balance. For example, everywhere outside the pipe each orbiting fluid particle is acted on by a balance of two forces as it performs its circular orbit in deep water: the outward centrifugal force and the inward pressure force. This is called the cyclostrophic balance (Kenyon, 1991). Within the pipe of small diameter there can be no orbital motion, but only motion in a straight line. Moreover, there is a mean pressure force that acts along this straight line to drive the flow.

It should be mentioned that Kepler invented a valveless water pump some 400 years ago (Caspar, 1993), but although I have not discovered the details of how this pump works, there is a strong probability that Kepler never had ocean applications in mind.

2. Wave Pump

Let us explore the physics of a different kind of wave water pump than the Isaacs one, the original motivation being not to explore the practical consequences of energy generation but rather to satisfy one's curiosity and understand how it works. (A practical application will nevertheless be mentioned later.) Basically, this is yet another example of Bernoulli's law in operation, but the circumstances are believed to be significantly different from all available applications, so that publishing a brief note about it is justified. The special conditions involve connecting two regions of flow in a wave field, where the type of wave is the surface gravity wave and the two regions are the one just beneath the wave surface—the active wave zone—and the other one at or below the depth of wave influence—the non-wave zone. A chief theoretical characteristic of surface gravity waves in deep water is that variations of pressure and velocity due to the wave

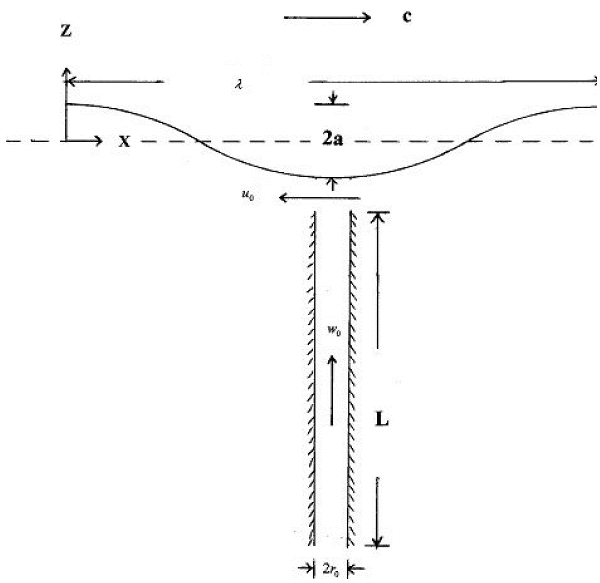

Fig. 1. One wavelength (λ) of a surface gravity wave of height ($2a$) is shown with a vertical piece of pipe fixed in position below the trough. Mean sea level is indicated by the dashed line. The wave travels in the positive x-direction with phase speed c, and the horizontal particle speed below the trough is u_0, which moves across the top opening of the pipe in the negative x-direction. Pipe dimensions are: length L and inside diameter $2r_0$, which is assumed to be much smaller than λ but still finite. Maximum steady vertical flow up the center of the pipe is w_0 in the positive z-direction.

decrease exponentially with increasing depth below the mean surface, so that for practical purposes the waves cannot be detected by instruments that measure *in situ* velocity or pressure at depths greater than or equal to about one half wavelength. We propose to span these two regions vertically with a single length of pipe.

When surface waves induce a mean vertical velocity up the pipe, there are two basic types of flow: laminar and turbulent. Even though the wave motion itself is always assumed to be laminar (i.e. no wave breaking), it could still cause turbulent motion inside the pipe for sufficiently large dimensions of the problem (e.g. sufficiently large Reynolds numbers). In the calculations that follow the Reynolds number will be assumed to be below the critical value for the onset of turbulence. In other words, the main focus here is on laminar pipe flow; discussions of turbulent pipe flow are left for the future.

Calculating the vertical velocity inside a submerged straight pipe when surface waves move past it involves a model and some assumptions. We adopt the steady laminar incompressible pipe flow model (e.g. Schlichting, 1968, which also discusses the time-dependent flows). One additional assumption, mentioned earlier, is that the diameter of the pipe is sufficiently small, compared to a wavelength, that the presence of the pipe does not significantly alter the properties of the wave. Also for simplicity, deep-water waves are assumed: the total depth of water is comparable to or exceeds a wavelength and the vertical scale of the pipe. Therefore, the equation of motion of the fluid inside the pipe reduces to the pressure gradient along the length of the pipe, balanced by the force of laminar friction

$$\nu\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}\right)=\frac{1}{\rho}\frac{\Delta p}{L} \tag{1}$$

where the friction term is on the left side, with the pressure gradient term on the right side: the pressure difference between the two ends of the pipe Δp divided by the pipe's length L and the constant density of the fluid ρ. In (1) the z-axis for the vertical velocity runs up the center of the pipe and the cylindrical coordinate r is measured in the horizontal plane perpendicular to this axis (Fig. 1). The vertical velocity, which is a function only of the radius of the pipe, is w and ν is the kinematic laminar viscosity coefficient. The hydrostatic balance of forces has already been eliminated from (1).

According to Bernoulli's law the magnitude of the pressure difference between the ends of the pipe is given by

$$\Delta p=-\frac{1}{2}\rho u^2 \tag{2}$$

where u is the horizontal flow speed at the top end of the pipe that is due to the wave motion. The flow rate across the bottom end of the pipe is assumed to be zero. At a fixed position near the surface the magnitude of the horizontal particle velocity due to a propagating surface gravity wave of infinitesimal amplitude, from the standard irrotational theory (Lamb, 1932), is

$$u=a\omega \tag{3}$$

where a is the wave amplitude and the wave frequency ω is related to the wave number k by the dispersion relation in the deep water limit

$$\omega^2=gk \tag{4}$$

g is the acceleration due to gravity and the phase speed c is given by $c = \omega/k$. The horizontal velocity (3) decays exponentially with increasing depth and for practical purposes $u = 0$ at a depth of about 1/2 wavelength. Taking u to be independent of time needs a brief justification, which is provided in the Discussion section.

Inserting (3) and (4) into (2) gives

$$\Delta p = -\frac{1}{2}\rho a^2 gk. \tag{5}$$

With (5) Eq. (1) is

$$\nu\left(\frac{d^2 w}{dr^2} + \frac{1}{r}\frac{dw}{dr}\right) = -\frac{gka^2}{2L}. \tag{6}$$

The only boundary condition for (6) is that the vertical velocity must vanish at the inside walls of the pipe; this is the no-slip requirement due to viscosity. In this steady state problem there are no initial conditions.

Equation (6) has the well-known parabolic profile solution for the vertical velocity

$$w = w_0\left(1 - \frac{r^2}{r_0^2}\right) \tag{7}$$

that satisfies the no-slip boundary condition at the inside wall of the pipe, where r_0 is the radius of the pipe, and the constant maximum vertical velocity in the center of the pipe w_0 is

$$w_0 = \frac{a^2 r_0^2 gk}{8\nu L}. \tag{8}$$

The value for the upward vertical velocity in (9) can be simplified slightly by taking the length of the pipe to be half a wavelength ($L = \lambda/2 = \pi/k$), which approximately equals the depth of wave influence. Then using the dispersion relation (4) also, (8) becomes

$$w_0 = \frac{g}{8\pi\nu}\left(akr_0\right)^2. \tag{9}$$

Therefore, if all other variables are held constant, the steady upward velocity is proportional to the square of the average wave "steepness" (ak). On the other hand, for a given wave steepness the upward flow increases as the square of the radius of the pipe. A numerical example is helpful. Choose $\nu = 10^{-2}$ cm^2/sec, $ak = 10^{-2}$ and $r_0 = 3$ cm, then from (9) we get $w_0 = 3.5$ cm/sec.

Laminar flow requires that the following constraint be placed on the variables

$$R_e = \frac{w_0 r_0}{\nu} = \frac{g}{8\pi}\left(\frac{ak}{\nu}\right)^2 r_0^3 \le 2300 \tag{10}$$

where the Reynolds number R_e is based on the maximum vertical flow at the center of the pipe, which is taken from (9), and the pipe's radius. The critical value of the Reynolds number for the onset of turbulence is normally taken as 2300, but this value is generally considered to be a minimum. Laminar pipe flow can occur for Reynolds numbers up to 10^4 or even higher if the inside of the pipe is very smooth (Faber, 1995).

The total volume rate of flow Q is a frequently presented quantity, and in this case it is

$$Q = \iint w(r)rdrd\theta = \frac{3\pi}{2} r_0^2 w_0 \tag{11}$$

where w_0 is given by (9), so as usual the volume flow is proportional to the fourth power of the radius of the pipe.

3. Discussion

Nothing has been said so far about the orientation of the openings of the pipe. This is of no consequence for the lower opening because it does not matter since no fluid velocity due to the waves moves across this opening; it is simplest to direct it straight downward. If the upper opening is pointed straight up, the pump will work well. But in this case the horizontal particle velocity of the wave moving back and forth over the opening will be time-dependent, as will the square of the horizontal velocity. The time average of the square of the horizontal velocity over a wave period is nonzero, however, so there will be a nonzero pressure deficit at the top end of the pipe according to Bernoulli's law, which will drive fluid up the pipe. In the example of a sinusoidal time variation, the only change that has to be made is that the right side of (5) needs to be multiplied by one half.

A slightly different configuration of the upper end of the pipe will lead to a more efficient pumping (more water per unit time). Bend the upper end of the pipe at right angles into the horizontal direction and choose that horizontal direction parallel to the crests and troughs of the wave. Then the wave particle velocity crossing the opening will be constant in magnitude although its direction changes continually throughout the wave period. The direction of the velocity makes no difference to Bernoulli's law. It is the constant magnitude of the velocity that makes the pump more efficient and is the reason for taking the right side of (5) independent of time in the first place.

A longer pipe than considered above with its lower end placed deeper in the water column than the depth of wave influence, while holding the top end in the same position, will still pump water but the flow will be weaker because the overall pressure gradient is lower, since the pressure deficit at the top stays constant while the pipe length is increased. On the other hand, making the pipe length shorter than the depth of wave influence is no advantage either, since the pressure difference between the ends of the pipe begins to decrease as the pipe length decreases. Of course, for a given length of pipe the most efficient pumping will occur when the upper end is as close to the trough level as possible without ever emerging into the air.

Suppose the upper end of the pipe becomes exposed to the air for half a wave cycle, for example. Rather than being fatal to the operation of the pump, its efficiency will only be halved because the vertical flow will then only occur during that half of the wave cycle for which the upper end of the pipe is submerged.

What if the pipe stays vertical with its upper end always below the troughs and the lower end at the depth of wave influence, but it is now allowed to drift along with the ocean currents? The pump will still operate well because in the ocean the wave speed normally greatly exceeds the speed of the currents. Therefore, a relative wave particle velocity will exist across the upper opening of the pipe as the pipe is advected by the currents. The pipe must also be long enough and have sufficient inertia so that it acts like the stable platform FLIP; i.e. it responds minimally to the vertical motion of the wave.

4. Application

An ocean application for the wave pump can be proposed similar to that of Maruyama *et al*. (2004) for their salt fountain: to bring nutrient rich water from depths up into the surface layers where the sunlight penetrates in order to increase biological productivity in the vicinity of the upper end of the pump. Two advantages of the wave pump over the salt fountain are: 1) the significantly faster pumping rate, and 2) the pump does not need priming to get it started. (An adaptation of the Isaacs pump to bring nutrient rich water to the surface has not been suggested so far to my knowledge.) In the numerical example given above the maximum vertical velocity of the wave pump is predicted to be about 3.5 cm/sec when the pipe radius is 3 cm, the length of the pipe is half a wavelength and the average wave steepness is about 1/100. That rate is an order of magnitude greater than the one quoted by Maruyama *et al*. (2004) of 2.45 mm/sec for the salt fountain with a pipe radius of 15 cm. If we had used the same radius of 15 cm, our pumping rate would have been $5 \times 5 = 25$ times larger than 3.5 cm/sec.

In our estimate the pipe length was taken to be half a wavelength. For swell with a wavelength of 100 m the pipe would then be 50 m long (or longer), whereas Maruyama used a pipe length of 280 m. If we used the same length of pipe (280 m) our upwelling speed would be reduced by a factor of about 50/280 or about 0.18 because of the reduced pressure gradient due to the greater length of pipe. Therefore, using the same radius and the same length for our pipe we get a pumping velocity of about 4.5 times 3.5 cm/sec, i.e. 15.6 cm/sec, which is nearly two orders of magnitude larger than that for the salt fountain. This is for 100 m wavelength swell with an average steepness of about 1/100.

Recently there has been an attempt to understand the greatly enhanced diffusivity of the salt fountain problem in field observations as compared to what was expected (Zhang *et al*., 2006), where the walls of the pipe are flexible and can oscillate in and out in response to the wave motion. Perhaps the present wave pumping mechanism could lead to an explanation of at least part of this enhancement.

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