End Wall Effects in Experiments on Water Intrusion along the Interface between Two Homogeneous Layers

Izumi TODA** and Yutaka NAGATA**

Abstract: End wall effect puts an inherent limitation on tank experiments, especially when the problems in a stratified fluid are dealt with. During experiments involving horizontal intrusion along the interface between two homogeneous layers, a curious phenomenon was found, i.e., the tip of the intruding water wedge continues to extend for a short time after supply is stopped, but then it begins to retreat in the cases of relatively high Reynolds numbers. The cause of this retreat of the wedge was investigated and was shown to be attributable to the initial disturbance generated near the mouth of the feeder at the start of water supply which propagates along the interface layer and reflects at the end of the tank as a bulge of the interface layer. The retreat of the intruding wedge would not occur in a sufficiently long tank, and so the cause of the retreat can be considered as one kind of end wall effect in a tank.

1. Introduction

In a previous paper written by MASUDA and NAGATA (1974), horizontal water intrusion along the interface between two homogeneous layers was investigated as one of the basic formation mechanisms of microstructures in oceanic thermoclines. It was found that the behavior of the intruding water wedge is strongly influenced by the detailed structure of the interface, and that the intruding wedge is virtually stopped several hours after water supply to the intruding wedge is cut off. The intruding velocity is slow and depends on “thickness” of the interface caused by diffusive action, even when the interface is sharp enough for its position to be visually determined. The time variation of the wedge length is found to be well described by the relation which is derived using an analogy to the case of intrusions into a linearly stratified medium. Though the deduced relation suggests that the intruding velocity is very small after water supply is stopped, it is still unaccountable why the intruding wedge achieves a stationary and balanced state since we cannot find any forces acting against the release of the potential energy.

In supplementary experiments with larger Reynolds number than in the previous cases, we found that the tip of the intruding wedge continues to extend for a short time after supply is stopped, but then it begins to retreat. Sometimes, the advance and retreat of the intruding wedge can be observed several times. With the hope that we may understand the reason of the smallness of the intruding velocity, we investigated this phenomenon experimentally. The conclusion obtained is disappointing in some sense, because the main cause of the retreat of the wedge is attributed to the initial disturbance generated near the mouth of the feeder at the start of water supply which propagates along the interface and reflects at the ends of the tank as a bulge of the interface layer or a solitary internal wave of the third mode in a three layered medium. The retreat of the intruding wedge would not occur in an infinitely long tanks, and the cause of the retreat can be considered as one kind of end wall effect in a tank.

However, the end wall effect itself is an important subject to be investigated because it gives an inherent limitation to tank experiments, especially when we deal with problems in a stratified fluid. The detailed generation mechanism of the initial disturbance is not discussed here but will be given in a separate paper.

2. Experimental procedure

We used a Lucite tank 29 cm deep and 7 cm
wide, the length of which was adjusted from 60 cm to 383 cm. Initially, we set two homogeneous layers in the tank. Fresh water was used for the upper layer throughout our experiments, and so its density $\rho_1$ was fixed to be 1.00 g cm$^{-3}$. Salt water was used for the lower layer, and its density $\rho_2$ was varied from 1.01 to 1.08 g cm$^{-3}$. The time interval between the first formation of the interface and the start of the supply of the intruding water was fixed at one hour in almost all experimental runs. If we take the diffusivity of the density due to salt diffusion to be $1.3 \times 10^{-4}$ cm$^2$ s$^{-1}$, thickness of the interface at start of the experiment is estimated to be about 0.9 cm (MASUDA and NAGATA, 1974). As to the thicknesses of the two layers $h_1$ and $h_2$, we examined the cases $h_1 = h_2 = 14$ cm and $h_1 = h_2 = 8$ cm. However, as we did not find any significant difference between these two cases, we set $h_1 = h_2 = 8$ cm in most of our experimental runs.

The schematic diagram of the experimental apparatus is shown in Fig. 1. The feeding system consists of two reservoirs, $A$ and $B$, and the water feeder $C$, which were connected by polynvinyl pipes to one another. The water was kept overflowing from the lower reservoir $B$ so that the water head $H$ was kept constant in each experimental run. The water of density $\rho_2 = (\rho_1 + \rho_2)/2$ supplied in this manner forms a sharp wedge at the level of the interface as shown in Fig. 1. The intruding water was coloured with brilliant blue. The length of the wedge $L$ was measured from the face of the feeder as a function of $t$, the elapsed time since the supply of the intruding water was started.

The experimental parameters are the tank length $L_0$, the density difference between two homogeneous layers $\Delta \rho = \rho_2 - \rho_1$, the water head $H$, and the length of the intruding water wedge $L$ at the time $T_0$ when the supply of the water to the wedge is stopped. The flow rate of the supplied water seems to be proportional to the water head $H$, as the flow inside of the narrow pipes should follow Hagen-Poiseuille's law. However, the flow rate was very sensitive to the diffuser stuffed in the feeder, and so flow rate was estimated from the averaged velocity $U$ of the tip of the intruding wedge for 20 cm $\leq L \leq 30$ cm. Reynolds number $Re = Uh/\nu$ was determined from this velocity $U$, the thickness of the intruding water wedge near the feeder $h$, and the viscosity of water $\nu = 0.01$ cm$^2$ s$^{-1}$. The value of $L$ was changed in the range 30 cm $\leq L \leq (L_0 - 20$ cm).

3. Retreat of the intruding water wedge after the supply of the water is stopped

Examples of the time variation of the wedge length $L$ for relatively high Reynolds numbers are shown in Fig. 2 for the case that $L_0 = 126$ cm. The time when water supply to the wedge is stopped is shown by a horizontal arrow for each experimental run. The increase of the wedge length is proportional to $t^{1/4}$ during the con-

![Fig. 1. Schematic diagram of the experimental apparatus. $A$ and $B$ indicate water reservoirs. The reservoir $B$ is kept overflowing so that the water head $H$ is constant throughout each experimental run. $C$ indicates the water feeder. We measure the length of the water wedge $L$ as a function of time $t$.](image)

![Fig. 2. Time evolution curves of the wedge length $L$. Reynolds number for each experimental run is shown at the righthand end of each curve. Curves with solid lines are the case for $\Delta \rho = 0.02$ g cm$^{-3}$ and those with dashed lines are the case for $\Delta \rho = 0.04$ g cm$^{-3}$. The time $T_0$ at which water supply is stopped is shown by a horizontal arrow for each curve.](image)
Continuous water supply as discussed by Masuda and Nagata (1974) and Maxworthy (1972). After water supply is stopped, the rate of increase of the wedge length decreases gradually, and then the tip of the wedge begins to retreat. In the cases shown in Fig. 2, we can recognize repetition of the advance and retreat of the intruding wedge two or three times. The period of the repetition (the time difference between the first and second maxima) decreases with increase of density difference as seen in Fig. 2. When the length of the water channel $L_e$ is fixed, the occurrence time of the first maximum $T_{\text{max}}$ depends both on the wedge length $L_s$ at the time $T_s$ and on the density difference $\Delta \rho$ (see Fig. 3 for notations). The period of the repetition is proportional to $(\Delta \rho)^{1/2}$ but does not depend on Reynolds number, and the amplitude, $L_{\text{max}} - L_{\text{min}}$, of the variation in the wedge length (the difference between the wedge lengths at the first maximum and at the minimum) is roughly proportional to Reynolds number but does not depend on $\Delta \rho$.

We find good correlation between the distance, $L_e - L_s$, to the end wall from the tip of the intruding wedge at $T_s$, and the time difference, $T_{\text{max}} - T_s$, between the time retreat starts and the time water supply is stopped as shown in Fig. 4. This relation seemed to be reasonable at first, because $(L_e - L_s)$ might represent the magnitude of deformation of the density structure in front of the intruding wedge due to internal waves preceding the wedge as discussed by Manins (1976) and $T_{\text{max}} - T_s$ might be a parameter characterising the force which resists the advance of the intruding wedge and causes the slowness of the intruding velocity.

However, as discussed in the following sections, we found finally that the good correlation shown in Fig. 4 is caused by limitations in the parameter ranges employed in our experiments.

4. Cause of the retreat of the intruding wedge

Another good correlation can be found between the occurrence time $T_{\text{max}}$ of the first maximum of wedge length and the characteristic length $L_s = 2L_e - L_{\text{max}} (= L_e + (L_e - L_{\text{max}}))$ where $L_e$ is the tank length and $L_{\text{max}}$ is the wedge length at $T_{\text{max}}$, and is shown in Fig. 5. Except for five cases which are marked by a, b, c, d, and e in the figure (these cases will be discussed later), data points are well arranged along a straight line which passes through the origin.

It is likely that some disturbance is generated near the mouth of the feeder at the start of water supply. This initial disturbance may travel along the interface in front of the wedge with much higher velocity than that of the tip.
of the intruding wedge. The disturbance would be reflected at the end wall of the tank, and come back towards the wedge. When the disturbance meets the tip of the intruding wedge, the movement of the wedge would be influenced by the disturbance, and retreat of the wedge might occur. Then, \( T_{\text{max}} \) and \( L_t \) can be interpreted as travel time and travel length of the initial disturbance until it meets the tip of the intruding wedge, and the gradient of the regression line of the data points in Fig. 5 should indicate the propagation speed of the initial disturbance.

The time evolution curves of wedge length for the five exceptional cases of a, b, c, d, and e are shown in Fig. 6. The time \( T_s \) at which water supply was stopped is shown by a horizontal arrow. For each of these five cases, the evolution curve before \( T_s \) is not smooth and a bend of the curve can be recognized as marked by \( \bullet \) in the figure. If these bends of the curve indicate the encounter time of the wedge tip with the initial disturbance, \( T_{\text{max}} \) should be replaced by the occurrence time of the bend of the curve. Such rearrangements were done for these cases and the rearranged data points are shown with a, b, c, d, and e respectively, in Fig. 7. Then, the first maxima of the time evolution curve of wedge length should indicate the second encounter time of the wedge tip with the initial disturbance travelling backwards, and so \( T_{\text{max}} \) should be correlated with another travel length defined by

\[
L_t = 3L_s + (L_s - L_{\text{max}}) = 4L_s - L_{\text{max}}
\]

for these cases. Such rearrangements were also done and the rearranged data points are shown as \( a' \), \( b' \), \( c' \), \( d' \), and \( e' \) in Fig. 7. It is remarkable that all data points in Fig. 7 are now well aligned along a straight line passing through the origin.

All occurrence times of the second and the

Fig. 6. Time evolution curves of wedge length for the five exceptional cases a, b, c, d, and e in Fig. 5. Identification of the case, Reynolds number, and tank length are shown at right-hand end of each curve. Horizontal arrow indicates the time \( T_s \) at which water supply is stopped. These evolution curves are not smooth before \( T_s \), and bends of the curves are recognized as marked by \( \bullet \).

Fig. 7. Correlation between \( T_{\text{max}} \) and \( L_t \) after rearrangements for the five exceptional cases. Data rearranged by two different methods are shown with a, b, c, d, and e, and \( a' \), \( b' \), \( c' \), \( d' \) and \( e' \), respectively. See the text for details.
third maxima of time evolution curves of the wedge length were checked in a similar manner, and were found to be well correlated with the corresponding travel length $L_t$ defined by

$$L_t = 2nL_0 - L_{\text{max}}$$

where $n = 2$ for the second maxima and $n = 3$ for the third maxima. These results strongly suggest that the retreat phenomenon of the wedge tip is relating to the initial disturbance which is generated near the mouth of the feeder at the start of water supply.

The gradients of the regression line in the correlation graph between $T_{\text{max}}$ and $L_t$ (or the propagation speed of the initial disturbance) are investigated for several values of density difference $\Delta \rho$, and shown in Fig. 8. While the speed of the tip of the intruding wedge is proportional to $(\Delta \rho)^{1/8}$, that of the initial disturbance is proportional to $(\Delta \rho)^{1/2}$.

5. Characteristics of the initial disturbance travelling ahead of the intruding wedge

In order to clarify the characteristics of the initial disturbance travelling ahead of the intruding wedge, we put many small grains of styrene resin into suspension near the interface in the tank and observed movements of the grains which happen to have settled at the level of the center of the interface layer.

Examples of the movements of the styrene floats are shown in Fig. 9 for the case where $L_0 = 253$ cm, $\Delta \rho = 0.02$ g cm$^{-3}$ and $Re = 31$. The horizontal displacements in cm of two styrene floats which were located at distances of 151.6 cm ($\bullet$) and of 245.7 cm ($\bigcirc$) from the feeder at the start of water supply are shown in Fig. 9a. In this case, the time $T_s$ and wedge length $L_s$ when water supply was stopped are 126 s and 49 cm, respectively, and the time $T_{\text{max}}$ and wedge length $L_{\text{max}}$ at the first maximum of the evolution curve of wedge length are 410 s and 69 cm, respectively. So, the positions of two floats are far from the tip of the intruding wedge. It can be seen that one of the floats ($\bullet$) begins to move at 110 s and the other ($\bigcirc$) at 180 s. The horizontal orbital velocity of the disturbance was deduced from the change of float position and is shown in Fig. 9b. The time change of the orbital velocity shows that the initial disturbance has a nature similar to that of a solitary wave. The float ($\bullet$) begins to retreat at about 300 s, suggesting that the front of the disturbance which was reflected at the end wall comes back to the float position at this time.

By fixing $\Delta \rho = 0.02$ g cm$^{-3}$, similar experiments were conducted for various ranges of tank length (from 190 to 383 cm), Reynolds number (from 30 to 194), wedge length at the time water supply is stopped (from 30 to 150 cm), and time interval between the first formation of the interface and the start of the experiments (from 1 to 1.5 h). The mean speed of the initial disturbance
estimated from 19 independent data are
\[ C = 1.44 \pm 0.11 \text{ cm s}^{-1} \]
for \( \Delta \rho = 0.02 \text{ g cm}^{-3} \). This value shows good agreement with that estimated from the gradient of the regression line of the correlation graph between \( T_{\text{max}} \) and \( L_t \) (1.41 cm s\(^{-1}\) for \( \Delta \rho = 0.02 \text{ g cm}^{-3} \)).

If we assume that the initial disturbance propagates as a bulge of the interface layer or an internal wave of the third mode in a three layered medium (TSUJI and NAGATA, 1974), we can calculate the thickness \( D \) of the interface layer from the estimated speed \( C \) of the disturbance as 0.4 cm. This value is considerably smaller than that estimated from diffusivity of salt and time interval \( T \) between the first formation of the interface and the start of the experiment (\( D = 0.9 \text{ cm for } T = 1 \text{ h and } D = 1.1 \text{ cm for } T = 1.5 \text{ h} \)) (MASUDA and NAGATA, 1974). Direct measurements of the density profile of the interface layer would be needed in the future experiments in order to clear up the cause of this discrepancy.

Typical amplitude of the orbital velocity of the initial disturbance is of the order of 0.1 cm s\(^{-1}\) as seen in Fig. 9b, and the amplitude of the vertical displacement of the interface as an internal wave of the third mode is of the order of 0.15 mm (see Fig. 10). The bulge of the interface layer due to the initial disturbance is usually too small to be observed by eye.

By comparing the amplitudes of the orbital velocity of the disturbance measured at two different positions, we can estimate decay rate of the initial disturbance. For the case of \( \Delta \rho = 0.02 \text{ g cm}^{-3} \), the disturbance reduces its amplitude to \( 1/e \) while it propagates over the distance of about 125 cm. In our experiments, no significant loss of energy is observed when the disturbance reflects at the end of the tank.

6. Qualitative explanation of the retreats and advances of the tip of the intruding water wedge

If the reflected initial disturbance plays an important role in the retreat of the intruding wedge, smooth and monotonous evolution curves of the wedge length should be obtained in a tank of sufficient length. In Fig. 11, the uppermost curve shows the time evolution curve for the case that \( L_0 = 383 \text{ cm}, Re = 136, \text{ and } L_t = 80 \text{ cm} \). The initial disturbance reaches the end wall at \( t = 266 \text{ s} \), but it does not return to the wedge tip within the time range shown in the figure. So the curve is smooth and monotonous.

We observed the change of orbital velocity of the initial disturbance at the position near \( L = 215 \text{ cm} \), and then we calculated the probable time change of the water velocity due to the disturbance at the tip of the intruding wedge when the end wall is placed at \( L = 125 \text{ cm} \), by considering the known propagation speed and decay rate of the disturbance (we assume that the reflection coefficient at the end wall is 1).
The evolution curve of the wedge length for the case that $L_e=125$ cm is then estimated by linear superposition of the evolution curve for the case that $L_e=383$ cm on the probable displacement due to the initial disturbance, and is shown in Fig. 11 by a dashed curve. The lowermost curve in Fig. 11 indicates the time evolution curve actually observed in the tank having a length of 125 cm for the case with the same order of Reynolds number. The shape of the estimated curve bears a remarkable resemblance to that of the actual curve, except for a vertical displacement of about 15 cm.

The evolution curve for $L_e=383$ cm extends for more than 125 cm of the length of the shorter tank. So, the vertical shift seems to be inevitable, and this discrepancy should be attributable to another end wall effect, such as the change of the density profile between the wall and wedge tip when the wedge tip approaches near the wall, as discussed by MANINS (1976).

Another example of a comparison between the estimated evolution curve from the experiment using a longer tank ($L_e=383$ cm) and the actual evolution curve in a shorter tank ($L_e=125$ cm) is given in Fig. 12 for the case that $\Delta \rho=0.02$ g cm$^{-3}$ and $L_w=30$ cm. The amplitude of the oscillation of the curve is a little different due to the difference in Reynolds number, but the gross features of the curve are also very similar to each other in this case.

In the evolution curves in Fig. 12, a bend in the curve is seen near $t=1$ min as indicated by a vertical arrow. The occurrence time of this bend can be explained by considering that the information that water supply has ceased needs some time to propagate to the tip of the intruding wedge. The propagation speed of this information is estimated to be nearly equal to that of the initial disturbance.* Therefore, if we want to know the horizontal intrusion velocity for the case of no water supply, we should investigate the nature of the curve after the occurrence time of such a bend in the curve.

![Graph](image_url)

**Fig. 12.** Comparison of the actual evolution curve of the wedge length in a shorter tank with the probable curve estimated from the experiments in a longer tank. The evolution curve observed in the tank having length of 383 cm is shown by the uppermost curve as a solid line ($\Delta \rho=0.02$ g cm$^{-3}$ and $Re=154$). The dashed curve is the expected curve in a tank having length of 125 cm, deduced from the uppermost curve and the nature of initial disturbance observed in the longer tank. The lowermost curve shown as a solid line shows the evolution curve actually observed in the tank having length of 125 cm ($\Delta \rho=0.02$ g cm$^{-3}$ and $Re=58$). Horizontal arrow indicates the time at which water supply is stopped, and vertical arrow the time at which the information that water supply has ceased reaches the tip of the intruding wedge.

### 7. Conclusions and discussions

The phenomenon of the retreat and advance of the intruding water wedge observed after water supply is stopped can be explained by the orbital velocity of the initial disturbance which was generated near the mouth of the water feeder at the start of water supply. This disturbance is transmitted and reflected back and forth in the tank several times before it decays out due to friction.

The magnitude of the initial disturbance is very small and its amplitude of vertical displacement is of the order of a few tenths of a mm in our experiments. It should be noted that, though the bulge of the interface layer due to the disturbance is usually too small to be detectable by eye, it critically influences intrusion experiments along a sharp interface in tanks of limited length.

* This conclusion is somewhat curious because the thickness of the interface layer (namely, of the intruding wedge) in this case is evidently of the order of 1 cm. The slowness of this propagation speed is not yet understood.
The speed of the initial disturbance is proportional to square root of the density difference $\Delta \rho$, and it seems to propagate along the interface as a bulge of interface layer or as a solitary internal wave of the third mode in a three layered medium. Though the amplitude of the disturbance is roughly proportional to Reynolds number, the attempt to make the change of the volume of the water supply at the start be gradual is not so efficient to minimize the magnitude of the initial disturbance. Though the generation mechanism of the initial disturbance is not yet clear and requires further study, one of the possible mechanisms is as follows. At the first stage of the water intrusion just after the start of water supply, the front of intruding water may have a rounded shape, and a “head wave” is formed just as in the case of gravity currents or density currents. The “head wave” which propagates with the speed of density currents or of internal waves is separated sooner or later from the wedge-shape portion behind which propagates at a much smaller speed. This initial disturbance thus propagates far ahead of the wedge-shape intruding water. Follow up experiments are now in progress, and we hope that more detailed discussion can be given in the near future.

Besides the end wall effects due to the initial disturbance, we have other kinds of end wall effects as discussed in the section 6. It should be emphasized here that the consideration of end wall effects is essential for experiments in stratified media such as horizontal intrusion using a tank of limited length, as discussed in this paper.

Acknowledgements
The authors wish to express their thanks to Mr. Masao Fukasawa, Jiro Yoshida, Yoshiteru Kitamura and the other colleagues in the Geophysical Institute, University of Tokyo for their advice and discussion.

References

界面に沿った水の水平貫入実験でのエンドウォール効果について

戸田 いづみ*, 水田 豊*

要旨: エンド・ウォール効果は、特に成層流体中の現象をあつかう際には、実験結果に致命的な影響を与える。海洋の微細構造の成因に係わって二つの密度の異なる層の間の界面に沿って、貫入水の流れ形を調べる実験を行なった際、貫入水の供給を停止したあとしばらくして、レイノルズ数が比較的大きい場合に、貫入水の先端が一定速度で後退するという現象が見出された。しかし、この現象は、貫入水貫入の機構は直接関係せず、貫入水の供給の開始時に、供給口の近くで起こされた初期乱れが、界面層のふくらみの形で伝播し、貫入水の端で反射してきて、それが貫入水の先端の動きに影響することによって生じることが確認された。すなわちこの現象は、水槽の長さが十分長いと現れることが可能であり、エンド・ウォール効果の一つと考えることが出来る。