A Shipborne Wave-Recording System with Digital Data Processing*

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Abstract: A shipborne wave-recording system which consists of a sonic wave gauge, accelerometers, gyroscopes and a computer system is described. Signals from the measuring apparatus are fed directly into a shipborne digital computer system at a prescribed sampling rate. The time series of wave heights and the acceleration are transformed into Fourier series using an algorithm of Fast Fourier Transform. Errors contained in the observed wave heights due to ship motion are corrected in the Fourier series by using the Fourier coefficients for the vertical acceleration. Power spectra and waveforms can also be calculated in a short time with this system from Fourier coefficients. Examples of the observational results obtained in the central part of the East China Sea in 1969 are presented.

1. Introduction

Open-sea observations of sea waves have become increasingly important both for practical needs and for the basic study on the interaction between wind and sea waves. Though a ship is a useful platform for the open-sea observations, there is a serious problem: wave heights measured from a ship are subject to large errors due to ship motion. This paper describes a shipborne wave recording system based on a digital process developed by the authors at the Ocean Research Institute, the University of Tokyo, including software for the correction of errors due to ship motion.

In our wave recording system, signals from a wave gauge and an accelerometer are fed directly into a shipborne digital computer system at a prescribed sampling rate. Wave height is measured by a sonic wave gauge (Taira and Takeda, 1969) which is mounted on a bow-boom where ship’s interference to wave fields is generally minimized. To measure the vertical movement of the ship, an accelerometer of a strain-gauge type is attached to the wave gauge. The time series of wave heights and the vertical acceleration thus collected are transformed into Fourier series in a short time using an algorithm of Fast Fourier Transform (Coehran, et al., 1967). The Fourier coefficients of acceleration are then transformed into those of displacement (see next section). The Fourier coefficients of corrected waves are then readily obtained by adding the coefficients of raw waves to those of the displacement. The power spectrum of corrected waves can be calculated directly by summing squares of the synthesized Fourier coefficients over a certain frequency band. A time series of corrected sea waves can be produced by the inverse transformation.

The shipborne wave-recorders were reported by Tucker (1956) and Mark (1962). They used analog-type double-integrators to compute displacement of platform from acceleration. We had also tried double-integrations by an electric circuit of the same type, and found that there were large errors due to undesirable phase shifts of signals caused by the low-cut filter which was indispensable to avoid drift of the output signals.

The data processing with an analog-type processor has an advantage that the results can be obtained at the real time and that the information such as the maximum wave height can be read out directly. However, laborious work is necessary to calculate statistical characteristics of ocean waves, such as significant waves and power spectra. In contrast, our

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system is based on digital processing and the final results can be obtained in a short time after the data collection. Besides, the digital processing necessary for the calculation can be made even with a shipborne mini-computer.

2. Procedure of analysis

The direct measurement of the vertical displacement due to ship motion is hard to achieve in the open sea where no rest frame is available. However, the energy of the ship motion is concentrated generally in a narrower frequency band than about 0.1 Hz. Therefore, the measurement of the acceleration of the motion is relatively easy. The displacement can then be calculated by the double-integration of the acceleration. In this section, we describe numerical methods of the double-integration and the procedure of the error correction.

Consider a continuous function represented by Fourier series

\[
f(t) = \sum_{n=1}^{\infty} \left( a_n \cos nt + b_n \sin nt \right) \quad (1)
\]

The integral of the function \( f(t) \), \( F(t) = \int_0^t f(x) dx \), is obtained by integrating Eq. (1) term by term and we have

\[
F(t) = \sum_{n=1}^{\infty} \left( \frac{a_n}{n} \sin nt - \frac{b_n}{n} \cos nt \right)
+ \sum_{n=1}^{\infty} \frac{b_n}{n} \sin nt \quad (2)
\]

In the same manner, the double-integral \( G(t) = \int_0^t F(x) dx \) is

\[
G(t) = -\sum_{n=1}^{\infty} \left( \frac{a_n}{n^2} \cos nt + \frac{b_n}{n^2} \sin nt \right)
+ t \sum_{n=1}^{\infty} \frac{b_n}{n} + \sum_{n=1}^{\infty} \frac{a_n}{n^2} \quad (3)
\]

Let the function \( f(t) \) be the vertical acceleration of the ship motion, then \( F(t) \) and \( G(t) \) represent the vertical velocity and displacement, respectively. The second term of Eq. (3) represents a bias proportional to the time and the third represents the mean value. In the case of ship motion, these terms may be assumed to be zero and Eq. (3) is reduced to

\[
G(t) = -\sum_{n=1}^{\infty} \left( \frac{a_n}{n^2} \cos nt + \frac{b_n}{n^2} \sin nt \right) \quad (4)
\]

Although the above relation of the Fourier series is exact, it is not practical nor possible to obtain such an infinite series that could be determined only by an observation of infinite length of time. In practice we observe for a finite time length and obtain a discretely sampled series of finite length,

\[
X(j \cdot \Delta t) = f(t_j - j \cdot \Delta t); \quad j = 0, 1, 2, \ldots, N-1,
\]

where \( \Delta t \) is the sampling time interval. From the time series, we can determine a finite Fourier series of length \( N \)

\[
X(j \cdot \Delta t) = \sum_{i=0}^{N/2} \left( A_i \cos \frac{2\pi}{N} ij + B_i \sin \frac{2\pi}{N} ij \right) \quad (5)
\]

where the Fourier coefficients are

\[
A_i = \frac{2}{N} \sum_{j=0}^{N-1} X(j \cdot \Delta t) \cos \frac{2\pi}{N} ij, \quad i = 1, 2, \ldots, \frac{N}{2} - 1
\]

\[
B_i = \frac{2}{N} \sum_{j=0}^{N-1} X(j \cdot \Delta t) \sin \frac{2\pi}{N} ij, \quad i = 1, 2, \ldots, \frac{N}{2} - 1
\]

\[
A_0 = \frac{1}{N} \sum_{j=0}^{N-1} X(j \cdot \Delta t), \quad B_0 = 0
\]

\[
A_{\frac{N}{2}} = \frac{1}{N} \sum_{j=0}^{N-1} (-1)^j X(j \cdot \Delta t), \quad B_{\frac{N}{2}} = 0 \quad (6)
\]

If we select the number \( N = 2^\nu \), where \( \nu \) is an integer, the calculation can be made effectively by digital computers using an algorithm of Fast Fourier Transform (Cochran, et al., 1967).

Comparing Eq. (5) with Eq. (1), we have formally

\[
n = 2 \pi f_h = \frac{2\pi}{N \cdot \Delta t} i \quad (7)
\]
If the actual acceleration* of ship motion is expressed by a finite Fourier series of Eq. (5), then Eq. (1) can be reduced to Eq. (5) by using Eq. (7) and the displacement at the sampled time is readily obtained by Eq. (4) which can be written for our case as

\[ Z(j \cdot \Delta t) = G(t_j = j \cdot \Delta t) = - \sum_{i=1}^{N/2} \left( \frac{N \Delta t}{2 \pi i} \right)^2 \left( \frac{2}{N} \cos \frac{2\pi}{N} t_j + B \sin \frac{2\pi}{N} t_j \right) \] (8)

This equation, however, represents precisely the actual displacement only when the following assumptions are satisfied:

(i) Oscillations of frequencies higher than \(1/2 \cdot \Delta t\) as well as lower than \(1/N \cdot \Delta t\) are not contained in the actual acceleration.

(ii) Spectral structure of the acceleration takes a "line-spectrum" type and each frequency of the 'line' agrees with one of the \(f_i\)’s set (i.e., \(f_i; i=1,2,\ldots,N/2\)).

The first assumption will be satisfied with a good approximation by designing observations (i.e., by taking a suitable sampling interval and a sufficient length of time series), because the energy of the ship motion is concentrated mainly in a narrower frequency band. On the other hand, the second requirement is difficult to meet because the ship motion is inherently random and irregular, and the spectral structure is continuous.**

The errors due to the second source is the

* This does not mean the discretely-sampled time series of the vertical acceleration but the original continuous signal itself. The time series can be expressed strictly by a series of Eq. (5) and the Fourier coefficients are determined uniquely. In practice, round-off errors of our software appear at the last figure of ten significant digits (of a fixed point part in floating representation) after the inverse transformation is made.

** The energy of the ship motion is, sometimes, considered to be concentrated at resonant frequencies corresponding to its six motional degrees of freedom (i.e., pitching, rolling, yawing, surging, swaying and heaving). In our observations, however, multi-peaked power spectra were not observed as will be shown later. This fact suggests that the motion is mainly caused by direct actions of ocean waves.

Fig. 1. The leakage error of the finite Fourier transformation in a case where \(f_e = f_{60.5}\) and \(N=2048\).

"leakage error" of the Discrete Fourier Transform (MALING et al., 1967) for a continuous spectrum. The amount of the leakage error is illustrated in Figure 1 for a case where \(f(t) = 1.0 \cos (2\pi \times 60.5t), \Delta t = 1.0\), therefore, \(X(f) = 1.0 \cos (2\pi \times 60.5f),\) and \(N=2048\). For simplicity, following notations are used in the figure:

\[ |\tilde{X}(f)| = |\tilde{X}(f_i)| = \sqrt{A^2 + B^2} \]

\[ f_e = f_{60.5} \]

We define the \(f_i\) th coefficient, which is not actually calculated, by

\[ |X(f_i)| = \frac{1}{2} \left( \sqrt{A_{60}^2 + B_{60}^2} + \sqrt{A_{61}^2 + B_{61}^2} \right) \]

Here, \(f_i\) is none of the \(f_i\) 's set and lies be-
The leakage error is, on the other hand, not caused in a case where $f_c$ agrees with one of the $f_i$'s set. The spectrum is exactly a "line" as we examined.* Letting $t=2t'$, the signal $f(t) = 1.0 \cos(2\pi \times 60.5t')$ is expressed as $f(t') = 1.0 \cos(2\pi \times 121t')$ by the time series of length $2N$. In this case we have no leakage error caused by the frequency but the leakage error is still introduced by other frequencies to which coefficients are not actually calculated. The leakage error vanishes only for a time series of infinite length by which a continuous spectrum can be determined.

The double integration by Eq. (8) works as a "low-pass filter" which estimates coefficients of lower frequencies by multiplying larger weights. A broken curve in Figure 1 represents the doubly-integrated coefficients. A dashed curve shows their squares which correspond to

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* The leakage error was examined in eleven cases where $f(t) = 1.0 \cos(2\pi f_c t)$, and $c$ varied between 60.0 and 70.0 by a step of 0.1. In the two cases of 60.0 and 70.0, the leakage error was zero (i.e., $\Delta f_0$ or $\Delta f$ = 1.0, and the remaining coefficients were zero). The leakage error was the maximum in the case of $c=60.5$ which is plotted in Figure 1. The error, for example, in the case of $c=60.1$ was approximately one-hundredth as large as that of the case $c=60.5$. 

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Fig. 2. Examples of the observed power spectra: the vertical acceleration (left) and the raw wave height (right).
the power spectrum of the displacement. These two curves show that large errors are caused by the leakage error especially at the lowest frequencies and that there is a minimum near $f_0$ in each curve. In this study, these characteristics of the leakage error are applied to the double integration of the acceleration.

Here we consider the double integration of the observed time series. Figure 2 shows one of the observed power spectrum of the vertical acceleration due to ship motion (left). The spectrum has a sharp peak at the frequency 0.146 Hz that corresponds to $i=60$ in the finite Fourier series of length $N=2048$. The sampling interval was 0.2 second. The right graph shows the spectrum of the raw wave height. The peak of the graph is located at the same frequency but it is less sharp than that of the acceleration. These two spectra show that the ship motion and the ocean waves are closely related each other.

We assume, as a worst case, that the spectral peak of the vertical acceleration consists of signals whose frequencies are none of the $f_i$'s set. Then it would be better to select $i=40$ as a lowest frequency for the double integration. Otherwise, the error due to the double integral of the leakage error becomes very large at the lower frequencies as shown in Figure 1. Instead of Eq. (8), we have the displacement:

$$ Z(j \cdot \Delta t) = \sum_{i=1}^{N/2} W(i) \left( A_i \cos \frac{2 \pi}{N} ij + B_i \sin \frac{2 \pi}{N} ij \right) \quad (9) $$

where

$$ W(i) = \begin{cases} 0 & \text{for } i < 40 \\ \left( \frac{N \cdot \Delta t}{2 \pi i} \right)^2 & \text{for } i \geq 40 \\ \end{cases} $$

By using the calculated displacement $Z(j \cdot \Delta t)$, the time series of the corrected ocean waves is represented by

$$ \eta(j \cdot \Delta t) = \eta'(j \cdot \Delta t) + Z(j \cdot \Delta t), $$

where $\eta'(j \cdot \Delta t)$ is the time series of the raw wave height. The correction can be made in the Fourier space by firstly determining the Fourier series:

$$ \eta'(j \cdot \Delta t) = \sum_{i=0}^{N/2} \left( A_i \eta_i \cos \frac{2 \pi}{N} ij + B_i \eta_i \sin \frac{2 \pi}{N} ij \right) \quad (10) $$

and then

$$ \eta(j \cdot \Delta t) = \sum_{i=0}^{N/2} \left( A_i \eta_i + W(i) A_i \eta_i \cos \frac{2 \pi}{N} ij \\ + B_i \eta_i + W(i) B_i \eta_i \sin \frac{2 \pi}{N} ij \right) \quad (11) $$

Thus the corrected wave profile is readily obtained by the inverse Fourier transformation of Eq. (11). The power spectrum of the corrected waves can be calculated by summing squares of the synthesized Fourier coefficients over a certain frequency band (TAIRA, 1971).

3. Ship instrumentation

Figure 3 shows schematically the observational arrangement we made at the time of a first experiment in 1969 aboard the research vessel Hakuho-maru of the Ocean Research Institute, the University of Tokyo.* A sonic wave gauge was mounted on a tentative boom about one meter long. The response characteristics of our remote sensing wave gauge were previously reported by TAIRA and TAKEDA (1969). This gauge is able to measure with satisfactory accuracy ocean waves of wave length longer than about 6 meters (i.e., waves

![Fig. 3. Observational arrangement aboard the research vessel Hakuho-maru for the first experiment in 1969.](image)

* The research vessel has a housing on a boom projecting 10 meters from the bow at a height of 7.5 meters above the sea surface. The housing was not yet completed at the time of the observational period in 1969, however.
of about 2 seconds period in deep water). An accelerometer was mounted rigidly on the bowpole one meter apart from the wave gauge. Gyroscopes were set on the floor of a ship-laboratory about 40 meters apart from the bow to detect pitching and rolling angles of the ship motion. The analog signals from these measuring apparatus were fed to a shipborne computer (FACOM 270-20 with sixteen channels for analog-input) which was operated under the Real Time Control System.*

The accelerometer used in this study is of strain-gauge type (±1 g full scale) and its static characteristics are shown in Figure 4. In the upper portion, the output voltage is plotted against the inclination angle of a gymbal on which the accelerometer was mounted rigidly. Asymmetry of the curve was caused by the inclination of the gymbal at the initial setting (about 0.2°). The voltage is replotted in the lower portion against \( g \cos \theta \), where \( \theta \) is the corrected angle and \( g \) is the gravitational acceleration. The figure shows that the linearity of the accelerometer is satisfactory and that the accuracy is within 1 cm/sec\(^2\). Though the graph shows only the characteristics around the pitching axis, the accelerometer has the same characteristics around the other axis. The dynamic calibration is not yet made but it is expected that the accelerometer has good characteristics over a broad band of frequency from D.C. up

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* The software of the computer for our wave recording system had not been completed by the observation time, so the main calculations described in this paper were carried out by a computer (OKITAC-5090) at the Ocean Research Institute.

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Fig. 4. Static characteristics of the accelerometer. Output voltage against the inclination angle (upper) and that against \( g \cos \theta \) (lower).

Fig. 5. Wind data on 7th July, 1969 observed aboard in the central part of the East China Sea.
to about 10 Hz, because its natural frequency is high enough (20 Hz) and the damping ratio is 0.7.

In our observations, the accelerometer was rigidly mounted directly on the ship’s bow. The true vertical acceleration is represented by (see Appendix, Eq. A 12)

\[
V = X + g - A_x' \sin \theta + A_y' \frac{\sin \varphi}{\sqrt{1 + \tan^2 \theta}} + \frac{A_z'}{\sqrt{(1 + \tan^2 \varphi)(1 + \tan^2 \theta)}}
\]

where \( A_x' \), \( A_y' \), and \( A_z' \) are the components of the acceleration measured by accelerometers fixed on the ship. The angles (\( \theta \) and \( \varphi \)) represent the pitching angle and the rolling angle respectively. This relation represents the vertical acceleration \( X \) due to the ship motion.

In the observation which will be described in the next section, only \( A_x' \) was measured, and we regarded the apparent acceleration \( A_x' \) as the true vertical acceleration \( V \), (see also section 5).

4. Observational results

Observations were made on the research vessel Hakuho-maru which was anchored west off Kyushu in the central part of the East China Sea at 31°31.5'N and 127°00.0'E where sea depth was approximately 106 meters. Four observational runs were made from 17:47 to 19:12 (J.M.T) on 7th July in 1969 and each run took 409.6 seconds (= 0.2 second x 2048). Wind direction and speed, during the observational period, are shown in Figure 5. The data were collected by an anemometer of vane

![Graphs of X, Z, \( \eta' \), and \( \eta \) vs. Time (sec)]

Fig. 6. Examples of the waveforms: \( X \)… the vertical acceleration, \( Z \)… the displacement (the double integral of the acceleration), \( \eta' \)… the raw wave height, \( \eta \)… the corrected wave height.
type that was mounted on the foremost 21.8 meters above the mean sea level. A light wind of 4-5 m/sec from the direction of west or west-southwest was observed by 8 a.m. Then it decayed and calm prevailed until 12 a.m. After 12 a.m., wind speed increased as shown in Figure 5 and wind waves were developing. In the observational period, wind speed was about 10 m/sec and its direction was south or south-west.

Examples of observed waveforms are shown in Figure 6. They are of the first fifty seconds in the third run (begun at 18:45). The first and the third graphs show the signals from the accelerometer (X) and those from the sonic wave gauge ($\gamma'$), respectively. Double integration of the acceleration was carried out by using Eq. (9). The variation of $Z$ thus calculated is shown by the second graph of Figure 6. The figure clearly shows that waveform of the displacement is smoothed and that the phase relation is out-of-phase compared with the acceleration. Note also the phase relation between the displacement and the raw wave height: the raw wave height becomes higher when the ship's bow lowers. Waveform of the corrected ocean waves was calculated by using Eq. (11) and the result of $\eta$ thus calculated is shown by the fourth graphs.

When the motion of the center of gravity of the ship can be neglected, the vertical displacement of the ship's bow must be caused by the rotational motion of pitching or rolling. The rotational radius of the pitching motion was about 50 meters and that of rolling motion

Fig. 7. Computed vertical displacement (upper) and the pitching angle recorded simultaneously (lower).

Fig. 8. Pitching angle $\theta$ against the computed displacement. An arrow shows the angular resolution of the gyroscope (0.2°).
Fig. 9. Power spectra of the corrected ocean waves.
was about 7.5 meters. If these two rotational angles were of equal magnitude, the displacement was mainly associated with the pitching motion. The upper graph of Figure 7 shows an example of the vertical displacement and the lower shows the pitching angle, observed simultaneously. This is one of the parts where ship motion was the largest. Relationship between the pitching angle and the vertical displacement is plotted in Figure 8. A proportional relation is clearly seen. A solid line in the figure represents the relation \( \theta = \tan^{-1} \left( \frac{Z}{R} \right) \), where \( R \) is the distance from bow to the center of gravity (48 meters). This relation explains well the observed values.

The power spectra of the ocean waves thus corrected are plotted in Figure 9. As described before, their accuracies are good within the frequency range from the lower limit 0.098 Hz (due to the cutoff of the double integration) to the higher limit 0.5 Hz (caused by the response characteristics of the sonic wave gauge). The most striking fact in Figure 9 is that the spectral densities of the corrected waves do not decrease compared with those of the raw waves. There are some cases where their magnitude even increases up to 5% by the correction. This fact may seem to be somewhat strange but it is shown even in examples of short time span presented in Figure 6 that the amplitude of the waveform \( \eta \) produced by the correction is almost equal to that of raw wave \( \eta' \).

The four spectra in Figure 9 have peaks near the same frequency. The phase velocity of ocean waves corresponding to them is nearly equal to the wind speed during the observational period. This fact suggests that the spectra are of fully developed wind waves. A solid line in each graph represents the equilibrium spectrum suggested by Phillips (1966). Some observed spectral densities are two or three times as large as the equilibrium spectrum. This tendency of overshooting was also observed in the data obtained at a fixed marine tower.

5. Discussion

The analysis of waveforms indicates that the method of the correction of ship motion presented in this article is satisfactory and our wave recording system seems to be promising. However, the system at present form can measure with satisfactory accuracy ocean waves of limited frequency band only. The higher limit can be expanded up to about a few cycles per second by using a wave gauge of capacitance type instead of the sonic wave gauge. The lower limit, due to the cutoff of the double-integration, may be expanded to the lowest frequency by using a numerical filter “han” (Maleng et al. 1967). The filter diminishes the leakage error very effectively. However, the filter changes the waveforms to a large extent, the “han” is not suitable for determining the waveform of the corrected ocean waves.

In this paper, we regarded the signals from the accelerometer fixed on the ship as the true vertical acceleration. In the observations described above, both the angles of pitching and rolling were small (less than 2° as shown in Figures 7 or 8). The error due to ignoring the horizontal acceleration may be then 3 cm/sec² if the amplitude of the horizontal acceleration can be assumed to be equal to those of the vertical acceleration. This corresponds to an error of 3% as large as the vertical acceleration. The orthogonal projection of the gravitational acceleration to the apparent vertical axis is given by

\[
g/\sqrt{(1 + \tan^2 \varphi)(1 + \tan^2 \theta)}
\]

In this case, the error is less than 1 cm/sec² which corresponds to only 1% of error. When pitching and rolling angles are large, the horizontal accelerations due to ship-motion should be measured and the vertical acceleration should be computed according to Eq. (12).

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References


Appendix: On the coordinate system fixed on a ship

Consider a rectangular coordinate system xyz-α whose origin α is fixed on the ship's bow where measurements are carried out. The horizontal x-axis is taken to coincide with the ship's direction and y-axis is positive towards the port (Figure A). The vertical axis z is upward positive. Consider also another coordinate system x'y'z'-α which is fixed on the ship and has the same origin α. When the ship is at rest, the two system coincides with each other. Let the angles made by the axes x' and y' with the true horizontal axes x and y be θ (pitching) and ϕ (rolling), respectively. These angles represent those measured by a vertical-gyroscope which always keeps a hori-

zontal plane (i.e., xy-plane) and detects the angles between the plane and the axes of the frame (x'y'-axes) fixed on the ship.

To determine the relation between the two coordinate systems, we represent the axes by unit vectors

\[ X(1, 0, 0), \quad Y(0, 1, 0) \quad \text{and} \quad Z(0, 0, 1) \]

For the system x'y'z'-α, we assume the following forms:

\[ X' (\cos \theta, \sin \theta), \quad Y'(\alpha, \cos \varphi, \beta) \]

and \[ Z'(\gamma, \delta, \epsilon). \] (A1)

The sign of the pitching angle θ is defined to be positive when the ship's bow heaves. Note that the vector X intersects with the vector Y at right angles (i.e., X'·Y=0). By the ship's rotational motion around a vertical axes, the x-axes makes an angle ϕ with the x₀-axes of a coordinate system fixed on the earth (Figure A). The angle ϕ is measured by a directional gyroscope. When the measurements on shipboard of vectorial variables (e.g., directional spectra of ocean waves, velocity fluctuations in the air or water) are carried out, the relations among the three systems, x'y'z'-α, xyz-α and x'y'z'-α, are necessary for the correction of errors due to ship motion. On the other hand, for the measurements on the wave height which is a scalar variable, the relations between the two systems, x'y'z'-α and xyz-α, are sufficient to correct the error of the apparent vertical acceleration.

Five unknowns \( \alpha, \beta, \gamma, \delta \) and \( \epsilon \) in Eq. (A1)
are determined by the relations

\[ \alpha^2 + \cos^2 \varphi + \beta^2 = 1 \] (A 2)

\[ \gamma^2 + \delta^2 + \varepsilon^2 = 1 \] (A 3)

\[ \alpha \cos \theta + \beta \sin \theta = 0 \] (A 4)

\[ \gamma \cos \theta + \delta \sin \theta = 0 \] (A 5)

\[ \alpha \gamma + \delta \cos \theta + \varepsilon \beta = 0 \] (A 6)

The first two equations represent the conditions that vectors, \( Y' \) and \( Z' \), are unit vectors (i.e., \(|Y'|=|Z'|=1\)). The latter three represent conditions that each vector intersects at right angles with one another (i.e., \( X' \cdot Y' = X' \cdot Z' = Y' \cdot Z' = 0 \)). Solving Eqs. (A2-A6), we obtain

\[ \alpha = \frac{-\sin \varphi \tan \theta}{\sqrt{1 + \tan^2 \theta}} \] (A 7)

\[ \beta = \frac{\sin \varphi}{\sqrt{1 + \tan^2 \theta}} \] (A 8)

\[ \gamma = \frac{-\tan \theta}{\sqrt{(1 + \tan^2 \varphi)(1 + \tan^2 \theta)}} \] (A 9)

\[ \delta = \frac{-\tan \varphi}{\sqrt{1 + \tan^2 \varphi}} \] (A 11)

\[ \varepsilon = \frac{1}{\sqrt{(1 + \tan^2 \varphi)(1 + \tan^2 \theta)}} \] (A 11)

In Eq. (A8), the sign of the rolling angle \( \varphi \) is defined to be positive when the port heaves. The sign of \( \varepsilon \) (the orthogonal projection of the vector \( Z' \) to the vertical axes) is taken to be always positive; otherwise the ship will overturn.

Letting \( A_x' \), \( A_y' \), and \( A_z' \) be the components of the acceleration measured by accelerometers fixed on the ship, the true vertical acceleration \( V \) is described as

\[ V = A_x' \sin \theta + A_y' \frac{\sin \varphi}{\sqrt{1 + \tan^2 \theta}} + A_z' \frac{1}{\sqrt{(1 + \tan^2 \varphi)(1 + \tan^2 \theta)}} \] (A 12)


電子計算機を用いた船舶波浪計測システム

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要旨：本編式波高計、加速度計、ジャイロおよび電子計算機によって構成された波浪計測システムについて報告する。測器からの信号は船載電子計算機を用いて、所定のサンプリング間隔でデジタル化して記録する。得られた時系列をFFT法によってフーリエ級数で表現する。加速度のフーリエ係数を用いて船体の上下動の変位を求め、船から測定したみかけの波高に含まれる誤差を補正する。補正はフーリエ係数間で行ない、補正された波高のフーリエ係数から波浪スペクトルが算出される。フーリエ変換も容易に行うことができ、波形を求めることができる。

この計測システムを用いて、1969年に東京湾で観測したデータの解析結果もあわせて述べる。