Journal of the Oceanographical Society of Japan
Vol. 27, No. 1, pp. 1 to 6, February 1971

Reflection and Breaking of Internal Waves on a Sloping Beach*

Hideki NAGASHIMA**

Abstract: The reflection and breaking of internal waves on a sloping beach were studied in a small wavetank filled with water and petroleum. The dependence of the reflection coefficient of the internal waves on wave steepness and on beach slope is found to be very similar to that of surface waves. The reflection coefficient is small for the very gentle slope, increases rapidly as the slope increases, and becomes almost constant for the steep slope. The reflection coefficient decreases with increase of the wave steepness. Also, the transition slope at which the coefficient curve has the maximum gradient increases with increase of the wave steepness. Breaking pattern of the internal waves is classified into four types; "breaking", "semi-breaking", "wrinkle-generating", and "non-breaking". Their dependence on beach slope and wave steepness is examined. The regular sequence of the four breaking types from "breaking" to "non-breaking" is observed with decrease of wave steepness or with increase of beach slope.

1. Introduction

With the progress of oceanic instruments and measuring systems including towers and bouy stations, internal waves in the ocean have become familiar to oceanographers. However, detailed properties of their generation, propagation and dissipation remain unknown.

The behavior of internal waves entering a beach of a constant slope was investigated in a small wavetank filled with water and petroleum. Although only limited properties of oceanic internal waves of the lowest mode can be modeled in such a two-layer experiment, the use of the two immiscible fluids gives us the advantage that movement of the interface can be measured and recorded with a simple capacity-type wave-gauge.

2. Experiments

We used a Lucite wavetank 3.3 m long, 30 cm deep and 8 cm wide and examined internal waves at the interface between the water and the petroleum. The thickness of each layer was 5.0 cm throughout the experiments. We set a sloping beach at one end of the wavetank. A Tefron plate was used for the beach bottom to minimize energy absorption due to friction The slope of the beach was changed by 17 steps from 3/50 to the vertical. A plungertype wave-maker was placed at the other end. The plunger having an elliptical face moves up and down at the interface and generates internal waves. A wave-filter filled with film chips was located in front of the wave-maker to regulate the wave profiles and to reduce the influence of multireflection. The whole apparatus is shown schematically Fig. 1.

A wave of 1.9 sec. period was selected in the experiment for the reflection coefficient. For longer period waves, appreciable multireflection occurs and influences the determination of the wave envelope. For shorter period waves, appreciable wave attenuation occurs within a shorter distance than the beach length. In the experiment for breaking pattern, we used wave periods from 1.5 sec. to 2.5 sec. The amplitude of internal waves was changed by adjusting the stroke of the wave-maker, and measured by a

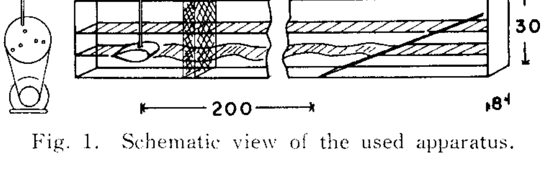

Fig. 1. Schematic view of the used apparatus.

---

* Received August 15, 1970

** Earthquake Research Institute, University of Tokyo

capacity-type wave-gauge. The sensor of the gauge consists of two parallel aluminum plates which are insulated with lacquer. Since the dielectric constant of petroleum is much smaller than that of water, movement of the interface can be measured by the change of the capacity between two plates. On the other hand, movement of the petroleum surface was measured by a pressure-type wave gauge.

3. Reflection coefficient of internal waves on a sloping beach

The incident and reflected internal waves create partially standing internal waves. The reflection coefficient R can be obtained by measuring the envelope of the partially standing waves (Healy's method);

$$R=\frac{E_{\max}-E_{\min}}{E_{\max}+E_{\min}},$$

where $E_{\max}$ and $E_{\min}$ are the maximum and minimum values of the envelope. If the effect of frictional damping is taken into account, the right-hand side of the above equation decreases with the increase of the distance from the beach. Then, denoting the attenuation constant by ν and the wavelength by λ, we have

$$\frac{E_{\max}-E_{\min}}{E_{\max}+E_{\min}}=Re^{-\frac{2\nu\bar{x}}{\lambda}},$$

where $E_{\max}$ and $E_{\min}$ must be evaluated for each pair of the adjoining maximum and minimum of the envelope, and $\bar{x}$ is the average distance of these maxima and minima measured from the beach. We measured the envelope of the partially standing internal waves at an interval of 5 mm over two wavelengths by moving the wave-gauge, and the reflection coefficient at the toe of the beach was calculated by evaluating the attenuation constant ν.

Reflection coefficients of 1.9 sec. internal waves versus beach slopes for three wave steepness, 1.8, 3.0 and 4.0 %, are shown in Fig. 2 through Fig. 4. The incident wave steepness is defined here as the ratio of the wave height at the toe of the beach to the wavelength over the flat bottom of the tank.

The reflection coefficients are small for very

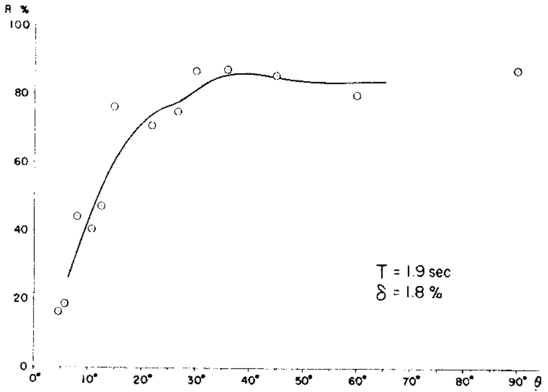

Fig. 2. Reflection coefficient R against beach slope angle θ in the case of 1.8 % wave steepness δ. ⦿ represents experimental data. The solid line shows smoothed curve obtained by taking three points moving average of experimental reflection coefficient.

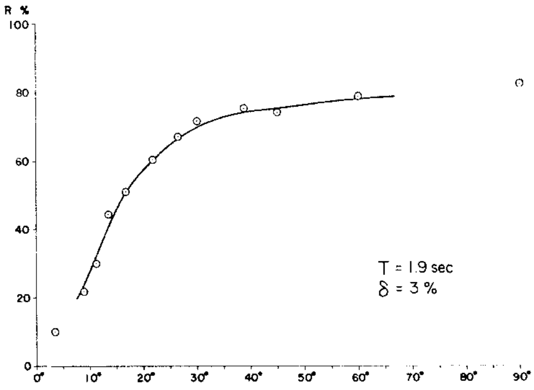

Fig. 3. Reflection coefficient R against beach slope angle θ in the case of 3.0 % wave steepness. The solid line shows smoothed curve.

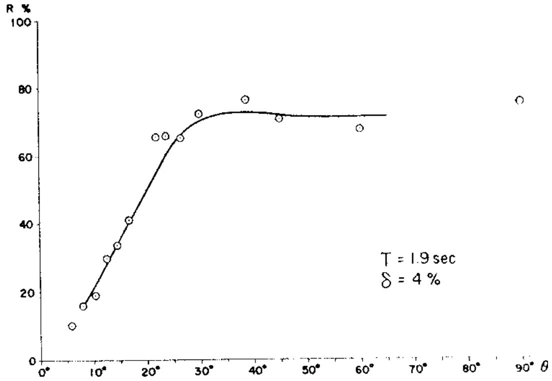

Fig. 4. Reflection coefficient R against beach slope angle θ in the case of 4.0 % wave steepness. The solid line shows smoothed curve.

gentle slopes, increase rapidly in the transition slope range, and are almost constant for large slopes. For a constant beach slope, the reflection coefficient decreases with increase of the wave steepness. If we represent the transition slopes as the slope angle of 50 % reflection, this characteristic angle increases with the increase of the wave steepness; the characteristic angle is 20.5° for 4.0 % wave steepness, 16.5° for 3.0 % and 12.0° for 1.8 %.

Many experimental works have been done about the reflection coefficient of surface waves on a sloping beach (*e.g.*, GRESLOU and MAHE, 1954). The reflection coefficient curves of the surface waves for 1.0 %, 2.0 %, 3.0 % and 4.0 % deep-water steepness are cited from Greslou and Mahe's paper and shown with our results for the internal waves in Fig. 5. The deep-water steepness* of the internal waves corresponding to the steepness of 1.8 %, 3.0 % and 4.0 % at the toe of the beach in our experiments are 1.6 %, 2.6 % and 3.5 %, respectively. The experimental results of the reflection coefficient of the internal waves show rough agreement with Greslou and Mahe's curves for the surface waves, and suggest that the mechanism of the energy loss of the internal waves over the sloping beach is very similar to that of the surface waves.

For gentle beach slopes, the reflection coefficients of internal waves are slightly larger than those of the surface waves. The large amount of the energy dissipation over the gentle beach should be attributed to wave breaking. The slight difference between the two reflection coefficients may result from the difference of breaking manner between the internal and surface waves; (1) the backward orbital flow of the upper fluid may restrain the foreward breaking motion of the lower fluid, (2) on the other hand, the large velocity shear at the interface may make internal waves more unstable, and (3) the relatively large effect of the interfacial tension in our experiment has the tendency to depress the sharpness of the breaking wave crests, and may restrain breakings.

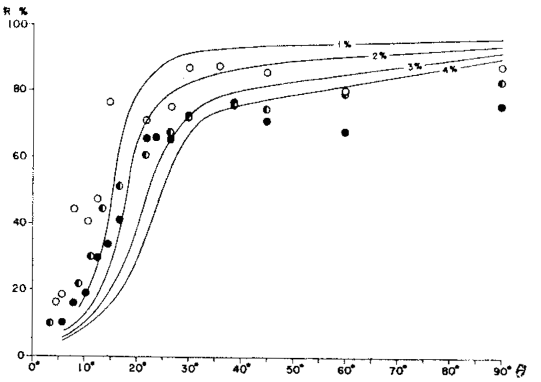

Fig. 5. Comparison of reflection coefficient of internal waves with those of surface waves. The reflection coefficient against beach slope angle of surface waves for 1.0 %, 2.0 %, 3.0 % and 4.0 % deep water steepness by solid lines (after GLESLOU and MAHE, 1954). The experimental results of the internal waves are shown by ○ for 1.6 %, by ◐ for 2.6 % and by ● for 3.5 % deep water steepness, respectively. These deep water steepness of internal waves correspond to 1.8, 3.0, 4.0 % wave steepness at the toe of the beach, respectively.

For large beach slopes, no breaking can be observed. However, the reflection coefficients of both internal and surface waves are considerably less than 100 % even in the case of the nearly vertical wall. This discrepancy may be interpreted as the accumulation of the experimental errors such as (1) frictional energy loss along side walls and on the bottom (and also along the interface in the case of internal waves) over the beach, (2) the existence of the higher frequency waves which were caused by inadequate shape and movement of the wavemaker, and (3) the incorrect use of the Healy method on the skewed shape of the finite amplitude waves. For the case of surface waves of finite amplitudes, GODA and ABE (1968) gave tne correction diagrams of Healy method. If we apply this correction to Greslou and Mahe's reflection coefficients curves, the reflection coefficients for slope angles larger than

---

* Consider an infinitely deep ocean of two layers; the densities of the upper and the lower layer are the same as those in the experiment, and the thickness of the upper layer is the same as in the experiment but that of the lower layer is infinite. The deep-water steepness of the internal waves is defined in this imaginary ocean as the steepness of the internal waves which have the same frequency and the same amount of energy flow as in the experiment.

50° reach 100% actually. Though the correction of the same type for internal waves is too complicated and impractical, the large portion of the apparent energy loss for large slopes in our experiments may also be attributed to the effect of the skewed shape of the finite amplitude waves in the used Healy method.

Throughout our experiment, the records of the pressure-gauge do not show any appreciable waves of surface mode. The energy exchange between waves of surface and internal modes is negligible in our experiment.

4. Breaking and other energy absorption mechanism

As discussed in the previous section, the large energy absorption occurs in the case of gentle beach slopes. Internal waves break clearly over the gently sloping beach, though the manner of breaking is somewhat different from that for the surface waves; the wave crest is less sharp due to relatively large interfacial tensions, but the shape of waves at the front of crests is deformed into very irregular shape and then many foams of petroleum are generated there. These foams are sometimes carried offshore by orbital currents of the upper fluid, which flows in the opposite direction of that of the lower fluid.

The behavior of the waves over the sloping beach are observed with a movie camera, and the dependence of the breaking type on wave steepness and on the beach slope are examined. The wave behavior near the beach is classified into four breaking types as follows;

1) "breaking" when waves break and many foams are generated near the crest. The foams are blown off offshore by the orbital currents in the upper fluids. (See Fig. 6)

2) "semi-breaking" when the deformation of the wave crest looks like breaking, but no stable foams are generated.

3) "wrinkle-generating" when the drastic deformation is not seen, but short capillary waves are generated near the shoreline at the time when the incident wave crest reaches there. The generated capillary waves are recognized to propagate offshore. (Fig. 7)

4) "non-breaking" when no noticeable deformation occurs.

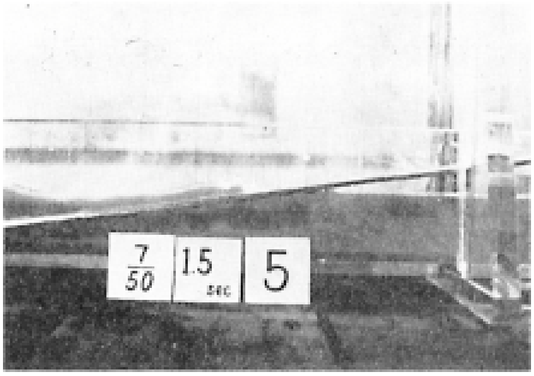

Fig. 6. Typical breaking of the internal waves. Many foams are generated near shore line. Period: 1.5 sec., beach slope: $^{7}/_{50}$, wave steepness at the toe: 4.2%.

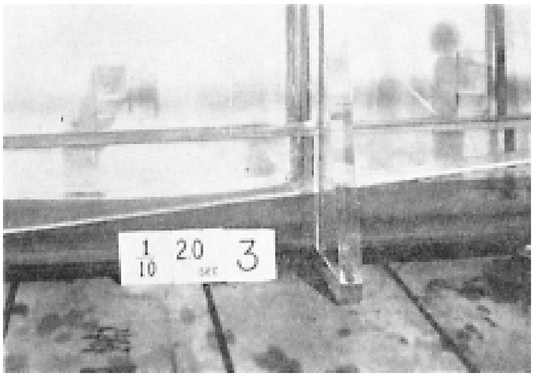

Fig. 7. Breaking of "wrinkle-generating" type. No drastic deformation of the wave crest occurs, but the short capillary waves are generated near shore line and propagate offshore. Steepness: 1.7%

The occurrence condition of these four breaking types for 2.0 sec. internal waves is shown in Fig. 8, where the wave steepness measured at the toe of the sloping beach is taken in the ordinate and the slope angle in the absissa. The classification of the breaking types is arbitrary and the details of wave deformation over the slope is changeable with change of uncontrollable experimental conditions. The results in Fig. 8 may be incorrect in detail. However, we can see the regular sequence of the breaking type from (1) to (4) with decrease of the wave steepness for a fixed slope, or with increase of the slope angle for a fixed wave steepness.

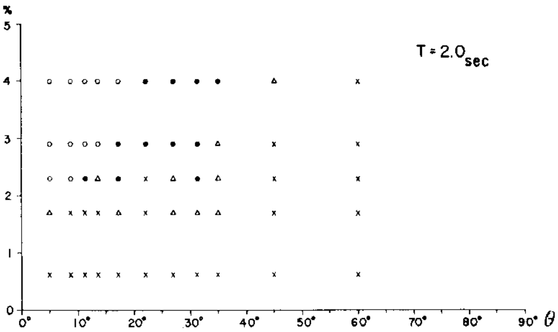

Fig. 8. The occurrence condition of breaking types depending on wave steepness δ and on the beach slope θ in the case of 2.0 sec. period. ○: "breaking", ●: "semi-breaking", △: "wrinkle-generating", ×: "non-breaking".

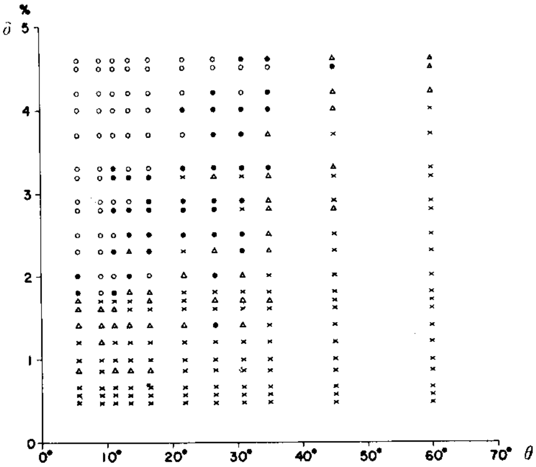

Fig. 9. The occurrence condition of breaking types depending on wave steepness δ and on the beach slope θ in the case of 1.5 sec., 1.8 sec., 2.0 sec. and 2.2 sec. period altogether. ○: "breaking", ●: "semi-breaking", △: "wrinkle-generating", ×: "non-breaking".

We made the same experiments for 1.5 sec., 1.8 sec. and 2.2 sec. waves. The results are plotted altogether in Fig. 9. The breaking type does not depend on the wave period at least in the range from 1.5 sec. to 2.2 sec. and within our experimental errors. In Fig. 10 summarizes the occurrence condition of breaking types to the wave steepness and the slope angle. The characteristic slope angles (the angles of 50 % reflection) discussed in the previous section are also shown in Fig. 10. The characteristic angles roughly fall in the "semi-breaking" domain.

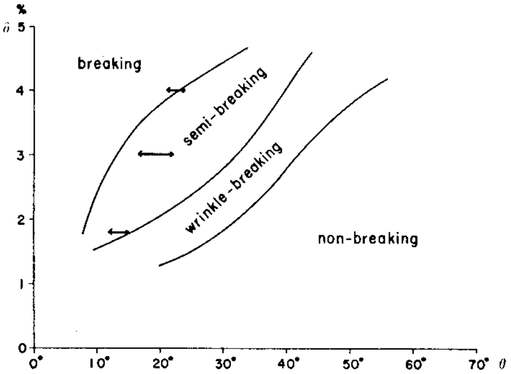

Fig. 10. The occurrence condition of breaking types depending on wave steepness δ and on the beach slope θ deduced from Fig. 9. The characteristic slope angles of 50-70 % reflection coefficients are also shown by horizontal line.

5. Energy of the wrinkles travelling offshore

TAIRA *et al.* (1968) studied the reflections of surface waves on a sloping beach in a very small wavetank. They also observed the capillary waves which were generated near shoreline and which propagate offshore under certain conditions. They pointed out the possibility that energy transfer to the capillary waves had a noticeable effect on the reflection coefficient curve. The similar energy transfer was observed in "wrinkle-generating" domain in Fig. 9. We shall check here whether such energy transfer affects the reflection coefficient curve or not. When the internal waves of 40 cm wavelength and of 1 cm waveheight approached the beach of 35° slope, we observed that from 7 to 8 wave trains of the offshore capillary waves of about 7 mm wavelength were generated. The energy per wavelength of the internal gravity waves E_g is given by*

$$E_g=\left[\frac{1}{4}(\rho_2-\rho_1)ga^2 + \frac{a^2\sigma^2}{4k}(\rho_1\coth kh_1+\rho_2\coth kh_2)\right]\cdot\frac{2\pi}{k},$$

where ρ_1 and ρ_2, are densities of the upper and lower fluids, h_1 and h_2 are thicknesses of the upper and lower layers, and a, σ, k and g are wave amplitude, angular frequency, wave number, and acceleration of gravity, respectively.

---

* For simplicity, rigid surface is assumed.

On the other hand, the energy per wavelength of short capillary waves E_c is given by

$$E_c=\left[\frac{1}{4}Ta^2k^2+\frac{1}{4}(\rho_2-\rho_1)ga^2+\frac{a^2\sigma^2}{4k}(\rho_1+\rho_2)\right]\cdot\frac{2\pi}{k},$$

where T is interfacial tension. Then for the example under consideration the energy per wavelength of the incident waves is 4×10^3 erg, and the energy per wavelength of capillary waves is about 8 erg as the observed amplitude is roughly 1 mm. So, even if 10 wave trains of the capillary waves are generated for each incident wave crest, the energy of the capillary waves travelling offshore is only a few percents of the incident internal gravity waves. Therefore, the energy transfer to the capillary waves seems to give little influences on the reflection coefficient curve.

Acknowledgements

The author wishes to express his sincere thanks to Professors Kozo YOSHIDA, Kinjiro KAJIURA, Yutaka NAGATA, Mizuki TSUCHIYA and Keisuke TAIRA for their suggestions and discussions.

References

GODA, Y. and Y. ABE (1968): Apparent coefficient of partial reflection of finite amplitude waves. Rept. of Port and Harbour Res. Inst., **7**(3), 3-58.

GRESLOU, L. et Y. MAHE (1954): Etude du coefficient de reflexion d'une houle sur un obstacle constitute par un plan incline. Proc. 5th Conf. on Coastal Eng.

TAIRA, K. and Y. NAGATA (1968): Experimental study of wave reflection by a sloping beach. Jour. Oceanogr. Soc. Japan, **24**(5), 242 252.

内部波の斜面上での砕波，および反射

長　島　秀　樹

要旨　内部波の斜面上での砕波，および反射についての研究を小型水槽を用いて行なった．実験は，水と石油による二層モデルで行なった．内部波の反射率の波形勾配と斜面勾配に対する依存性は，表面波のそれとよくにていることがわかった．反射率は，ゆるやかな斜面では小さく，斜面勾配が大きくなるにつれて急激に増大し，大きな斜面勾配のところではほとんど一定になる．又，波形勾配が大きくなるにつれて反射率は減り，反射率が小さい値から大きい値へ急激に変化する斜面勾配の範囲は，大きい斜面勾配の方へ移行することがわかった．

内部波の砕波を観察し，四つの型に分類した．それらは，“breaking”，“semi-breaking”，“wrinkle-generating”，“non-breaking” であり，これら四つの型の現象が，斜面勾配の増加，および波形勾配の減少とともに順次見られた．