Measurements of Vertical Air Temperature Gradients with Dipmeter*

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Abstract: By means of measuring the dips of horizon with a Pulfrich dipmeter, vertical air temperature gradients near the sea surface were obtained with an accuracy of ca. 0.01°C/m, much better than a conventional method with an array of temperature sensors. The additional advantage of this technique was that the measurements were made on board a ship underway, consequently data from wide oceanic areas could be accumulated rather quickly and easily. From the results of dip measurements at 125 stations on two cruises of the T/V Oshoro Maru of the Hokkaido University, extending from the Bering Sea to the south of Australia, the temperature gradients were computed with a new formula with an assumption that the refractive index of air varied with a height only. It was found that, in the northern North Pacific Ocean, vertical air temperature gradients were positive, while in the subtropical and tropical Pacific Ocean both the negative and positive gradients were observed. Generally, in the same sea region, the temperature profiles had the similar form, irrespective of air-sea temperature differences.

1. Introduction

The vertical temperature gradients near the sea surface are one of the important elements for air-sea interaction and studies have been carried out by various authors. For most of these studies, the measurements were made with the array of temperature sensors installed on a pole attached to a floating buoy or a ship. The accuracy of temperature measurement is, in general, within the order of 0.1°C, and that of temperature gradient is about 4×10⁻²°C/m, with a height difference of 5 m. Roll (1965) has emphasized the importance of measuring more accurately the temperature gradient near the sea surface and stated that "this accuracy could be improved by one order of magnitude if optical refraction measurements were employed for the determination of temperature and humidity gradients in air as shown by Brocks." By means of measuring the refraction of light ray, Brocks (1940, 1954) has pointed out that the mean temperature and humidity gradients over a long distance could be determined more accurately. With a suitable theodolite and a suitably designed mark, the accuracy of temperature gradient determination would be 5×10⁻⁴ °C/m with a distance of 15 km. The measurements were made at high mountains for temperature gradients of atmosphere, several kilometres in altitude, and later near the sea surface at the Elbe mouth.

Since the classical work of Koss (1900), empirical formulae estimating the dip of horizon with the height of eye with a correction due to air and surface water temperature difference were presented by various authors. Freiesleben (1950) studied the problem theoretically and gave a new formula in which a refractive coefficient \( k = \frac{n}{R} \left( \frac{dn}{dR} \right) \) was introduced, where \( R \) is the radius of the earth, \( h \) height and \( n \) the refractive index of air.

A handy instrument for measuring the dip of horizon was invented by Kohlschütter (1903) when he was working up the results of Koss’ observations. The instrument is named “Pulfrich dipmeter” according to its constructor. The dipmeter has a smaller accuracy than a good theodolite, but has an advantage that the measurements can be undertaken on board a ship underway. Therefore, with a dipmeter, a large number of data from a wide area of open

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oceans could be collected rather easily, and this would contribute to the study of air-sea interaction in a global scale.

Two of the co-authors, HYUGA and SAKAMOTO, made the observations on the dips of horizon with a Pulfrich dipmeter on board the T/V Oshoro Maru during her Cruise 16 to the South Pacific Ocean, from November 1965 to February 1966 and also Cruise 21 to the Bering Sea from June to August 1968. The observations were carried out very carefully, and their results could be used for estimating temperature gradients above the ocean, although the main purpose of their study was to introduce a new formula of estimating the dip of horizon.

2. Methods of observations

The method of observations was more or less the similar for both the cruises, there were, however, some improvements for Cruise 21; the method employed for the later cruise is described in this section.

The dips of horizon were measured at five different places of the ship, i.e. the upper deck, the boat deck, the wheel deck, the compass bridge and the top of the rader mast (not for Cruise 16). The heights of eye, about 3.8, 4.3, 6.0, 8.2, 10.5 and 15.7 m respectively, were determined up to one centimetre with corrections according to the draught of the ship.

The measurements of the dips of horizon were carried out three times a day, so far as the horizon was visible, around 09.00, 12.00 and 18.00 of LMT. The Pulfrich dipmeter has a minimum scale division of 0.1 minutes of arc, and by means of vernier scale, it was read to 0.01' of arc. The image of the horizon fluctuates around its mean position due to seintilations and also rolling and pitching of the ship. An arithmetic mean of ten measurements were taken, thus the dips of horizon were expressed in terms of 1/1,000 minutes of arc, its accuracy being estimated to be an order of 0.01 minutes of arc.

Dry and wet bulb temperatures were measured at the same heights of dip measurements simultaneously with an Assmann's type aspiration psychrometer. There were also carried out ordinary meteorological observations, including surface water temperature with bucket, wind force (in Beaufort scale) and direction, sea and swell, weather, atmospheric pressure, wet and dry bulb temperature in a shelter.

3. Calculation of temperature gradient

The refraction of ray near the earth's surface has been studied as a problem of geodesy. Generally the problem was treated under the assumption that ray is in the form of circular arc, but this assumption implies an assumption on a vertical temperature profile. In the present studies, it is assumed that the structure of atmosphere is horizontal, in other words, the refractive index of air is the function of height only, without any further assumption on the form of the ray.

The principle of geometrical optics says that the light ray takes the path of the minimum travel time, i.e.

$$\int \Delta t = \int \frac{ds}{v} = \int \frac{nds}{c} = \text{minimum}, \quad (1)$$

where $c$ is the velocity of light in vacuum; $n$, the refractive index of air; $\Delta s$, a length element.

A polar co-ordinate, which the origin lies at the centre of the earth, is taken, then Eqn. (1) is written

$$\int_{A}^{B} \frac{n(r)}{\sqrt{1 + r^2 \left( \frac{d\theta}{dr} \right)^2}} dr = \text{minimum}, \quad (2)$$

where $A$ and $B$ denote the horizon and the observation point, respectively. It is possible to consider that the earth is a sphere and the ray has a common tangent with the earth's surface at the horizon. Putting $R$ the radius of the earth and $h$ the height of the eye, then

$$\int_{\frac{R+h}{n(r)} \sqrt{1 + r^2 \left( \frac{d\theta}{dr} \right)^2}}^{\frac{R}{n(r)}} dr = \text{minimum}. \quad (3)$$

The principle of variation calculus leads the following differential equation.

$$\frac{d}{dr} \left( n(r) r^2 \theta / \sqrt{1 + r^2 \theta^2} \right) = 0$$

or $n(r) r^2 \theta / \sqrt{1 + (r \theta)^2} = C. \quad (4)$

Taking the point where the light ray touches the horizon as the axis of the co-ordinate, then, at $\theta = 0$, $\frac{dr}{d\theta} |_{\theta = 0} = 0$ and $r_\theta = 0 = R$. The integral constant is
\[ C = \lim_{n(r) \to 0} n(r) r^2 \theta' / \sqrt{1 + (r \theta')^2} = n_0 R, \quad (5) \]

where \( n_0 \) is the refractive index of air just above the sea surface. With Eqs. (4) and (5), we get

\[ \theta' = \frac{d \theta}{dr} = 1/r \sqrt{\frac{n_0 R}{n_0 R^2 - 1}}. \quad (6) \]

The dip of horizon is an angle between the ray and the circle \( r = R + h \), then

\[ \tan \delta = \frac{\Delta r}{r \Delta \theta} = \frac{1}{r} \left( \frac{d r}{d \theta} \right) = \sqrt{\frac{n_0 R}{n_0 R^2 - 1}}. \]

Since the dips of horizon are less than 6 minutes of arc or \( 2 \times 10^{-3} \) in radian or so, we can put \( \tan \delta \approx \delta \). Putting \( n = n_0 + \Delta n \), \( r = R + h \), we get, neglecting small quantities (10^{-6} against 1),

\[ \delta = \sqrt{\frac{2h}{R}} + \frac{2n_0}{n_0} \left( \Delta \theta^2 - \Delta \theta^2 \right), \quad (7) \]

or

\[ \Delta n = \frac{n_0}{2} \left( \Delta \theta^2 - \Delta \theta^2 \right), \quad (8) \]

where \( \Delta \theta = \frac{2h}{R} \).

The results of theoretical and experimental studies on the relationships between the refractive index of air and the atmospheric pressure, and humidity were summarized by the BROCKS (1934). The refractive index can be expressed

\[ (n - 1) \times 10^3 = a_1 \frac{p}{T} - a_2 \frac{e}{T}. \quad (9) \]

The coefficients \( a_1 \) and \( a_2 \) are functions of wave length. Somewhat different values were obtained by different authors, ranging from 78.8 to 79.5 and from 8.7 to 12.4 respectively, for the light of NaD-line, with the air temperature in \( ^\circ \)K, atmospheric pressure and vapour pressure in mb. 79 and 10 are used for the present studies, as generally accepted in the field of meteorology.

The accuracy of computing the vertical temperature gradient with an optical method depends on the accuracy of the above formula; it is considered, however, the values are sufficiently correct for our purpose, being an error of less than \( \pm 0.1\% \) for \( a_1 \). An approximate value is sufficient for \( a_2 \), since the effect of vapour pressure on the refractive index of air is relatively small.

Neglecting the second term in Eqn. (9), the difference of refractive indices at the height \( h \) and just above the sea surface, is

\[ \Delta n = 79 \left( \frac{\rho_h}{T_h} - \frac{\rho_0}{T_0} \right) \times 10^{-6}, \quad (10) \]

where the suffix \( o \) indicates the values just above the sea surface and \( h \), at the height of eye.

Equating (8) and (10) and introducing numerical values, we get,

\[ T_0 = T_h - (1 - 0.0342(h/T_h)) \]

\[ (0.000536 \Delta^2 - 0.000200h(T_h/p_0)) \quad (11) \]

and,

\[ T_h = T_0(1 - 0.0342(h/T_0)) \]

\[ (1 + (0.000536 \Delta^2 - 0.000200h(T_0/p_0)). \quad (12) \]

where \( \Delta \) is the dip of horizon in minutes of arc. Neglecting small quantities with a view of keeping the accuracy of computation within 1\%, the temperature difference \( \Delta T = T_h - T_0 \) can be expressed

\[ \Delta T = 0.0342h + (T_0/p_0) \times (0.000536 \Delta^2 - 0.000200h). \quad (13) \]

4. Accuracy of \( \Delta T \) determinations and the effect of vapour pressure

For calculating \( \Delta T \) with Eqn. (13), it is necessary to know the air temperature just above the sea surface, \( T_0 \), which has to be estimated with Eqn. (11) using the air temperature \( T_h \) at the height of measurement \( h \).

As it is stated in the previous section, \( T_h \) were measured at several heights at a station, an arithmetic mean of \( T_0 \) computed from \( T_h \) at different heights is used for calculating \( \Delta T \). Because of difficulties involved in air temperature measurements, the estimated values of \( T_0 \) are considered approximate, having relatively large standard deviations in some cases. As it is shown below, an approximate value is, however, sufficient to determine the temperature difference with an enough accuracy.

Differentiating Eqn. (13) with regard to \( T_0 \), we get

\[ \frac{\partial \Delta T}{\partial T_0} = 2\left( \frac{T_0}{p_0}(0.000536 \Delta^2 - 0.000200h) \right). \quad (14) \]

Considering the numerical values of \( p_0, T_0 \) etc., we can find that the values of \( \partial \Delta T/\partial T_0 \) is approximately 0.005, therefore even with an error of \( \pm 1^\circ \) in \( T_0 \), \( \Delta T \) can be determined with the accuracy better than \( \pm 0.01^\circ \)C.

The accuracy of various parameters in Eqn. (13), i.e., the atmospheric pressure, the height of eye and the dip of horizon are already dis-
cussed in the preceding section, we can expect that the temperature difference ∆T can be estimated within the error of ±0.02°C. Since the heights of measurements were more than 3 metres, the accuracy of the temperature gradients is estimated to be better than ±0.01°C/m.

The effect of vapour pressure gradient on the results of temperature gradient estimation as stated above needs more considerations. According to Brocks (1954), the vertical gradient of refractive index is, differentiating Eqn. (9),

\[
\frac{d}{dz} (\text{grad}^2 \text{mb}^{-2}) = \frac{d}{dz} (\text{mb}^{-1})
\]

<table>
<thead>
<tr>
<th>( t )</th>
<th>( \frac{d}{dz} (\text{mb}^{-1}) )</th>
<th>( \text{grad}^2 \text{mb}^{-2} )</th>
<th>( d_0 ) (mb⁻¹)</th>
</tr>
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<tr>
<td>0</td>
<td>3.60</td>
<td>1.05</td>
<td>( \times 10^{-6} )</td>
</tr>
<tr>
<td>10</td>
<td>3.34</td>
<td>0.99</td>
<td>( \times 10^{-6} )</td>
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<tr>
<td>20</td>
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<td>0.91</td>
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<tr>
<td>30</td>
<td>2.93</td>
<td>0.85</td>
<td>( \times 10^{-6} )</td>
</tr>
</tbody>
</table>

Fig. 1. Showing vertical air temperature gradients in the Pacific Ocean, from the Kuroshio region to south of Australia, on Cruise 16 of the Oshoro Maru.
\[
\frac{\partial n}{\partial h} = d_1 - d_2 \frac{p}{1000} - d_3 \frac{\partial T}{\partial h} - d_4 \frac{\partial e}{\partial h}. \tag{15}
\]

The numerical values of \(d_1\), \(d_2\), \(d_3\) are given by BROCKS and are reproduced in Table 1.

As it can be seen from the table, the values of \(d_1\) are very small and those of \(d_2\) are practically the same for dry and saturated air; therefore, only the last term in Eqn. (15) may have some effects on the estimated values of \(\Delta T\) with Eqn. (14).

This might be particularly so for the measurements during the Cruise 16 in the tropical region where the absolute humidity is high. Studies on the humidity gradients, e.g. TAKAHASHI (1962), show that, except in the lowest layer of a few metres, the humidity gradient is generally small and its effect on the refractive index gradient could be neglected. The corrections are, therefore, applied only for the temperature differences between the lowest height of observation and just above the sea surface. Assuming that the humidity of air just above the sea surface is in equilibrium with the sea water, the corrections are calculated by the following formula,

\[
\Delta T = \Delta T' + (1000\partial h/\partial x)(e_h - e_0), \tag{16}
\]

where a prime ' indicates a temperature difference with a humidity gradient, and \(e_h\) and \(e_0\) are the vapour pressure at the lowest height of observation and just above the sea surface, respectively. The accuracy of this correction is hardly estimated, it is, however, considered that an approximate value of vapour pressure is sufficient for the present purpose because of the small numerical coefficient in Eqn. (16).

5. Results

The temperature differences \(T_h - T_h = \Delta T\) against the height, computed with Eqn. (12), are illustrated in Fig. 1 and Fig. 2, taking its origin at the mean value of \(T_h\). The surface water temperature \(T_w\)'s are also indicated. From the figures, it is noticed that the temperature

Fig. 2. Showing vertical air temperature gradients in the northern North Pacific Ocean, on Cruise 21 of the Oshoro Maru.
difference between the air and the sea surface is not the main factor determining the temperature gradient, but each sea region seems to have its characteristic temperature profile. In order to illustrate this point more clearly, the geographical distributions of temperature gradients are shown in Fig. 3 and Fig. 4; in the figures, the gradients at the lowest layer are classified into the following categories:

- **positive gradient** \( \frac{dT}{dh} > 0.05^\circ\text{C}/\text{m} \)
- **small positive gradient** 
  \( 0.05^\circ\text{C}/\text{m} > \frac{dT}{dh} > 0.01^\circ\text{C}/\text{m} \)
- **neutral** 
  \( 0.01^\circ\text{C}/\text{m} > \frac{dT}{dh} > -0.01^\circ\text{C}/\text{m} \)
- **small negative gradient** 
  \( -0.01^\circ\text{C}/\text{m} > \frac{dT}{dh} > -0.05^\circ\text{C}/\text{m} \)
- **negative gradient** 
  \( -0.05^\circ\text{C}/\text{m} > \frac{dT}{dh} \)

In general, the temperature gradients were positive in the northern North Pacific Ocean during Cruise 21, and negative gradients predominated in the tropical Pacific Ocean and the Kuroshio region during Cruise 16. In the sea south of Australia, the gradients were very small. Since the two cruises were undertaken in the different seasons, it may be difficult to conclude that the results were regional or seasonal.
It is interesting to note that, even very wide varieties in areas, as well as in seasons, the lowest layer over the ocean is in a near neutral condition. Therefore, the vertical eddy flux of sensible heat could be estimated through the measurement of wind velocity and temperature gradients. The data are, however, not sufficient to discuss further on this point.

6. Conclusion

By means of measuring the dips of horizon, air temperature gradients in the lowest layer of several metres above the ocean could be obtained with an accuracy much better than an ordinary method with thermometers. The dips of horizon can be measured on board a ship underway with an instrument relatively simple and cheap, a dipmeter. The results of analysis indicate the feasibility of this technique. Should more data are collected, a map showing the distribution of vertical flux of sensible heat over the ocean could be constructed. For conducting the observation, particular attention should be paid for the determination of the height of eye. Considering the requirement for the accuracy of the height of eye, strict attention on the difference between individual observers has to be paid.

References


眼高差測定器による気温の鉛直傾度の測定

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要旨 眼高差測定器を用いて海面付近の気温の鉛直傾度を通常用いられる方法によるものより高精度（約 0.01°C/m）で測定した。この方法のもう一つの利点は、外洋を航行中の船上から測定出来るから、短期間に、かつ容易に広い外洋から多くの資料が集められることにある。北海道大学練習船おしょろ丸の2回の航海中で得られた、ベーリング海からオーストラリア南方海域にわたる125地点の眼高差の測定値を用いて，気温の鉛直傾度を計算した。この計算において，空気の屈折率が海面以上の高さだけの関数であるという仮定だけで計算するための式を導いた。

北太平洋北西部では気温の傾度は正，亜熱帯および熱帯太平洋では正も負も観測された。一般に同一の海域では気温と水深との間に関係なく気温の傾度分布は同じ型を示すことが観測された。