Earth Planets Space, **61**, 1163–1168, 2009

Calculation of the piezomagnetic field arising from uniform regional stress in inhomogeneously magnetized crust

Ken'ichi Yamazaki

Disaster Prevention Research Institute, Kyoto University, Gokasyo, Uji, Kyoto 611-0011, Japan

(Received January 21, 2009; Revised May 22, 2009; Accepted June 28, 2009; Online published November 30, 2009)

This paper describes a simple procedure for the calculation of the piezomagnetic field arising from uniform regional stress in heterogeneously magnetized crust. There exists a strong similarity between the spatial distributions of anomalies in the geomagnetic total force values arising from magnetization structures in the Earth's crust and those arising from piezomagnetic signals that arise from there. This similarity enables us to compute the piezomagnetic field due to uniform regional stress without the need to determine the explicit structure of magnetization intensities in the crust. This situation is similar to that of "reduction to the pole", which is commonly used to interpret magnetic survey data. An explicit formula is presented that gives the 2-D spectrum of the piezomagnetic field from that of local magnetic anomalies; the formula is then applied to synthetic data. Calculated values are compared with the exact solution of the piezomagnetic field in order to assess the efficacy of the proposed method. The comparison verifies that calculations performed using the formula yield sufficiently accurate values for practical use.

Key words: Piezomagnetic effect, regional stress, magnetic anomaly, reduction to the pole.

1. Introduction

The piezomagnetic effect describes the changes that occur in the magnetization of ferromagnetic minerals when these are subjected to mechanical stress. If we establish a quantitative relation between stress sources within the Earth's crust and changes in the magnetic field associated with these sources, we could then monitor tectonic processes via observations of the geomagnetic field. In fact, several studies have attempted to detect stress changes in the Earth's crust via measurements of the geomagnetic field, based on the piezomagnetic effect (e.g., Oshiman *et al.*, 1991; Stuart *et al.*, 1995). However, one important aspect of the piezomagnetic effect remains poorly understood. Experimental studies have revealed that changes in magnetization are proportional to changes in the applied stress when the stress is the same order of magnitude as those in the Earth's crust: i.e., up to several tens of megapascals (e.g., Nagata, 1970); the proportional coefficient in this case is referred to as the "stress sensitivity". However, piezomagnetic changes calculated based on stress sensitivity obtained via experimental studies are often underestimatings compared with observed values (Sasai, 1991, 2001; Nishida *et al.*, 2004). Until this discrepancy is resolved, it will remain difficult to conduct quantitative discussions on piezomagnetic field observations. Reliable values of *in situ* stress sensitivity are obtained by comparing simulated values, based on the piezomagnetic effect, and those obtained by actual observations. Therefore, the establishment of a method for estimating the piezomagnetic field is as important as making accurate observations of piezomagnetic signals.

The piezomagnetic field is generated in both the homogeneously and heterogeneously magnetized crust. In calculations involving the homogeneously magnetized crust, a representation theorem derived by Sasai (1980, 1991) plays an important role in simplifying the procedure followed in the calculation. Using the representation theorem, the piezomagnetic field can be calculated without the need to determine the explicit distribution of the stress field within a medium. Previous studies have obtained theoretical values using actual fault models to compare the values with observed changes in the geomagnetic field (e.g., Johnston *et al.*, 1994; Sasai and Ishikawa, 1997). In contrast to the relative simplicity of calculations for homogeneous magnetization, calculations of the piezomagnetic field arising from heterogeneously magnetized crust are problematic. Although the representation theorem is also applicable to the heterogeneously magnetized crust (e.g., Nishida *et al.*, 2007), in such cases the explicit distribution of the magnetization structure in the crust must be determined in advance, which is generally difficult to achieve. Nevertheless, studies on the heterogeneously magnetized crust are important because heterogeneity of the magnetization is one of the factors that act to enhance piezomagnetic signals (Oshiman, 1990).

An important situation to consider is that of uniform regional stress applied to the heterogeneously magnetized crust. Nishida *et al.* (2004) calculated the piezomagnetic field in just such a case in order to compare the result with observed secular changes. In this earlier study, the magnetization structure in the investigated area is estimated based on the results of a dense magnetic survey, and the piezomagnetic field is calculated by a forward simulation. Although this approach is complicated—and difficult to apply in many regions—there exists a similarity between magnetic anomalies and the piezomagnetic field if we only consider the uniform regional stress field (as described below). By taking this similarity into account, we can calculate the piezomagnetic field that arises from a medium without the need to determine the explicit distribution of magnetization within the medium. In this paper, we explore the nature of this similarity between magnetic anomalies and the piezomagnetic field in order to derive a representation that connects magnetic anomalies for a certain magnetization structure and the piezomagnetic field associated with the regional stress applied to the structure.

2. Piezomagnetic Field due to Regional Stress

To establish a simplified procedure for calculating the piezomagnetic field, a brief summary of the general procedure is provided in such a way that it reveals the similarity between magnetic anomalies and the piezomagnetic field. The new method is then proposed.

2.1 General procedure

Piezomagnetic calculations generally start from the constitution law given by Sasai (1980, 1991) to express magnetization changes at the location $\boldsymbol{x}'$, $\boldsymbol{J}_{\mathrm{p}}(\boldsymbol{x}')$ arising from the piezomagnetic effect:

$$\boldsymbol{J}_{\mathrm{p}}(\boldsymbol{x}') = \frac{3}{2}\beta\,\boldsymbol{\sigma}(\boldsymbol{x}')\,\boldsymbol{J}_{\mathrm{a}}(\boldsymbol{x}')\,, \tag{1}$$

where β, $\boldsymbol{\sigma}$, and $\boldsymbol{J}_{\mathrm{a}}$ are stress sensitivity, the deviatoric stress tensor, and the initial value of magnetization under zero stress, respectively. The magnetic potential due to the piezomagnetic effect, W_{p}, at an arbitrary point, $\boldsymbol{x}$, is calculated by integrating the contribution from the magnetization elements given by (1):

$$W_{\mathrm{p}}(\boldsymbol{x}) = \iiint \boldsymbol{J}_{\mathrm{p}}(\boldsymbol{x}') \cdot \nabla' \frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|} dV'(\boldsymbol{x}')\,, \tag{2}$$

where ∇' represents the gradient operator with respect to the variable $\boldsymbol{x}'$. On the other hand, the magnitude of variation in the geomagnetic field due to the piezomagnetic effect is generally much smaller than that in the ambient main field; consequently, the variation due to the piezomagnetic field, ΔF_{p}, is represented by $\Delta F_{\mathrm{p}} = -\boldsymbol{l} \cdot \nabla W_{\mathrm{p}}$, where $\boldsymbol{l}$ represents a unit vector parallel to the ambient geomagnetic field and ∇ represents the gradient operator with respect to the variable $\boldsymbol{x}$. Combining this with (2), we obtain the expression

$$\Delta F_{\mathrm{p}}(\boldsymbol{x}) = -\boldsymbol{l} \cdot \nabla \iiint_{V'} \boldsymbol{J}_{\mathrm{p}}(\boldsymbol{x}') \cdot \nabla' \frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|} dV'(\boldsymbol{x}')\,. \tag{3}$$

The distribution of initial magnetization, $\boldsymbol{J}_{\mathrm{a}}(\boldsymbol{x}')$, must be given to perform calculations using (1) and (3); however, this value is generally unknown. Thus, $\boldsymbol{J}_{\mathrm{a}}(\boldsymbol{x}')$ should be estimated before the piezomagnetic field is calculated using (3). The magnetic potential, $W_{\mathrm{a}}(\boldsymbol{x})$, which corresponds to $\boldsymbol{J}_{\mathrm{a}}(\boldsymbol{x}')$, is represented in the same form as that for the piezomagnetic effect:

$$W_{\mathrm{a}}(\boldsymbol{x}) = \iiint_{V'} \boldsymbol{J}_{\mathrm{a}} \cdot \nabla' \frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|} dV'(\boldsymbol{x}')\,, \tag{4}$$

and anomalies in the total force values are represented by

$$\Delta F_{\mathrm{a}}(\boldsymbol{x}) = -\boldsymbol{l} \cdot \nabla \iiint_{V'} \boldsymbol{J}_{\mathrm{a}}(\boldsymbol{x}') \cdot \nabla' \frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|} dV'(\boldsymbol{x}') \tag{5}$$

where V' represents the entire volume of the Earth's crust. In most studies concerned with the estimation of magnetization structures in the Earth's crust, the directions of magnetization are assumed to be uniform (e.g., Nishida *et al.*, 2004). Therefore, $\boldsymbol{J}_{\mathrm{a}}(\boldsymbol{x}')$ can be represented as

$$\boldsymbol{J}_{\mathrm{a}}(\boldsymbol{x}') = J_{\mathrm{a}}(\boldsymbol{x}')\,\boldsymbol{n}_{\mathrm{a}} \tag{6}$$

with $J_{\mathrm{a}}(\boldsymbol{x}')$ being a scalar function and $\boldsymbol{n}_{\mathrm{a}}$ being a constant unit vector. In addition, $\nabla'(1/|\boldsymbol{x}-\boldsymbol{x}'|)$ in the integrand of (5) is replaced by $-\nabla(1/|\boldsymbol{x}-\boldsymbol{x}'|)$. Replacement of ∇' by ∇ enables us to move the derivative operator outside the integral. Therefore, by substituting (6) into (5), we obtain the following form:

$$\Delta F_{\mathrm{a}}(\boldsymbol{x}) = (\boldsymbol{l} \cdot \nabla)(\boldsymbol{n}_{\mathrm{a}} \cdot \nabla) \iiint_{V'} J_{\mathrm{a}}(\boldsymbol{x}') \frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|} dV'(\boldsymbol{x}')\,. \tag{7}$$

The distribution of $J_{\mathrm{a}}(\boldsymbol{x}')$ is estimated in such a way that (7) accurately explains the observed anomaly.

2.2 Simplified procedure

The simplified procedure is obtained by rewriting $\boldsymbol{J}_{\mathrm{p}}(\boldsymbol{x}')$ in a form in which its direction and amplitude are explicitly expressed. In the case of $\boldsymbol{J}_{\mathrm{a}}(\boldsymbol{x}')$ described in (6), $\boldsymbol{J}_{\mathrm{p}}(\boldsymbol{x}')$ is rewritten as

$$\boldsymbol{J}_{\mathrm{p}}(\boldsymbol{x}') = \alpha(\boldsymbol{x}')\,J_{\mathrm{a}}(\boldsymbol{x}')\,\boldsymbol{n}_{\mathrm{p}} \tag{8}$$

where

$$\alpha(\boldsymbol{x}') = \frac{3}{2}\beta\,|\boldsymbol{\sigma}(\boldsymbol{x}')\,\boldsymbol{n}_{\mathrm{a}}|\,, \tag{9}$$

and

$$\boldsymbol{n}_{\mathrm{p}}(\boldsymbol{x}') = \frac{\boldsymbol{\sigma}(\boldsymbol{x}')\,\boldsymbol{n}_{\mathrm{a}}}{|\boldsymbol{\sigma}(\boldsymbol{x}')\,\boldsymbol{n}_{\mathrm{a}}|}. \tag{10}$$

When the regional stress is spatially uniform, $\boldsymbol{\sigma}$ and therefore $\boldsymbol{n}_{\mathrm{p}}$ do not depend on the location, $\boldsymbol{x}'$. Therefore, a similar consideration to that which leads to (7) can be used to derive the following expression for changes in the total force field values due to the piezomagnetic effect:

$$\Delta F_{\mathrm{p}}(\boldsymbol{x}) = \alpha\,(\boldsymbol{l} \cdot \nabla)\,(\boldsymbol{n}_{\mathrm{p}} \cdot \nabla) \iiint_{V'} J_{\mathrm{a}}(\boldsymbol{x}') \frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|} dV'. \tag{11}$$

Because the differential operators $\boldsymbol{l} \cdot \nabla$ and $\boldsymbol{n}_{\mathrm{p}} \cdot \nabla$ are commutative, (7) and (11) lead to the following equation:

$$(\boldsymbol{n}_{\mathrm{a}} \cdot \nabla)\,\Delta F_{\mathrm{p}} = \alpha\,(\boldsymbol{n}_{\mathrm{p}} \cdot \nabla)\,\Delta F_{\mathrm{a}}. \tag{12}$$

The relation (12) is the fundamental relation that enables us to obtain a simple procedure to calculate the piezomagnetic field for a uniform regional stress.

The explicit formula that gives ΔF_p is obtained by representing (7) and (11) as 2-D Fourier integrals:

$$\Delta F(x, y, z) = \iint \exp\left\{i\left(k_x x + k_y y\right)\right\} \cdot \Delta G\left(k_x, k_y; z\right) dk_x dk_y, \quad (13)$$

where x, y, and z represent coordinates in the northward, eastward, and downward directions, respectively; k_x and k_y are the wave-numbers with respective to x and y; ΔF represents ΔF_a or ΔF_p; ΔG represents the 2-D Fourier transform of ΔF. Since the magnetic potential, W, satisfies the Laplace equation, $\nabla^2 W = 0$, and because changes in the total force values are represented by $\Delta F = \boldsymbol{l} \cdot \nabla W$, ΔF should also satisfy

$$\nabla^2(\Delta F) = 0. \quad (14)$$

In the frequency domain, the above equation is written as

$$\frac{\partial^2}{\partial z^2}\Delta G\left(k_x, k_y; z\right) = \gamma^2 \Delta G\left(k_x, k_y; z\right), \quad (15)$$

where

$$\gamma = \sqrt{k_x^2 + k_y^2}. \quad (16)$$

Therefore,

$$\Delta G\left(k_x, k_y; z\right) = \exp\{\gamma(z - z_0)\} \Delta G\left(k_x, k_y; z_0\right). \quad (17)$$

Mathematically, not only (17) but also $\Delta G(k_x, k_y; z) = \exp\{-\gamma(z - z_0)\}\Delta G(k_x, k_y; z_0)$ is a solution of (15); however, such a term violates the physical requirement that the magnetic field arising from sources in the Earth's crust should converge to zero at an infinite distance from the Earth's surface. Using (17), (13) can be written as

$$\Delta F(x, y, z) = \iint_S \exp\left\{i\left(k_x x + k_y y\right) + \gamma(z - z_0)\right\} \cdot \Delta G\left(k_x, k_y; z_0\right) dk_x dk_y, \quad (18)$$

where S represents the entire 2-D plane.

Based on (12), (13), and (17), the relation between ΔG_a and ΔG_p is obtained as follows:

$$\Delta G_\mathrm{p}\left(k_x, k_y; z_0\right) = A\left(k_x, k_y; \boldsymbol{n}_\mathrm{a}, \boldsymbol{n}_\mathrm{p}\right) \Delta G_\mathrm{a}\left(k_x, k_y; z_0\right), \quad (19)$$

with a transfer function defined by

$$A\left(k_x, k_y; \boldsymbol{n}_\mathrm{a}, \boldsymbol{n}_\mathrm{p}\right) = \alpha \frac{i\{k_x(n_\mathrm{p})_x + k_y(n_\mathrm{p})_y\} + \gamma(n_\mathrm{p})_z}{i\{k_x(n_\mathrm{a})_x + k_y(n_\mathrm{a})_y\} + \gamma(n_\mathrm{a})_z}. \quad (20)$$

There are two cases in which the denominator of (20) becomes zero: (1) $k_x = k_y = 0$ or $k_x(n_\mathrm{a})_x + k_y(n_\mathrm{a})_y = 0$, and (2) $(n_\mathrm{a})_z = 0$. In the former case, we can define $A\left(0, 0; \boldsymbol{n}_\mathrm{a}, \boldsymbol{n}_\mathrm{p}\right)$ as zero because uniform magnetizations generate no change in the magnetic field. In the latter case, in contrast, we can avoid the situation by considering two distributions of initial magnetization, $\boldsymbol{J}^+\left(\boldsymbol{x}'\right)$ and $\boldsymbol{J}^-\left(\boldsymbol{x}'\right)$, both of which satisfy $\boldsymbol{J}^+\left(\boldsymbol{x}'\right) + \boldsymbol{J}^-\left(\boldsymbol{x}'\right) = \boldsymbol{J}_\mathrm{a}\left(\boldsymbol{x}'\right)$, $\left(\boldsymbol{J}^+\left(\boldsymbol{x}'\right)\right)_z \neq 0$, and $\left(\boldsymbol{J}^-\left(\boldsymbol{x}'\right)\right)_z \neq 0$. Because the magnetic fields due to two sources can be superposed, the sum of the piezomagnetic fields generated by $\boldsymbol{J}^+(\boldsymbol{x}')$ and $\boldsymbol{J}^-\left(\boldsymbol{x}'\right)$ gives that generated by $\boldsymbol{J}_\mathrm{a}\left(\boldsymbol{x}'\right)$.

Note that the concept that underlies the above procedure is essentially the same as that behind polar reduction, wherein the virtual total force anomaly at the North Pole is calculated (Baranof, 1957), as frequently used in preliminary analyses of aeromagnetic survey data. The difference here is that a scalar factor, α, is employed, and that the direction of magnetization after transformation is not restricted to an upward vertical direction.

In practice, the measured total force distribution is not identical to the magnetic anomaly that arises from the static magnetization structure of the Earth's crust. Magnetic measurements also incorporate the piezomagnetic field due to the stress field at the time of the observation. To take into account the piezomagnetic field in magnetic measurements, the transfer function A in Eq. (20) should be modified accordingly. Nevertheless, intensities of the piezomagnetic fields are generally much smaller than those of magnetic anomalies. The ratio of piezomagnetic signals to magnetic anomalies is approximately given by α given by (9), which depends on $\boldsymbol{\sigma}$ and β. In the Earth's crust, $\boldsymbol{\sigma}$, which is the deviatoric stress tensor, is up to several megapascals, and β is no larger than several 10^{-2} MPa^{-1}. Thus, α is as small as 10% of the magnetic anomaly. Therefore, as a first approximation it is reasonable to ignore the piezomagnetic field in the measured values; hence, the proposed procedure based on the transfer function (20) remains valid.

Another problem in applying the proposed procedure is the calculation of $G(k_x, k_y)$, the Fourier transform of $F(x, y)$. In real cases, dense magnetic-anomaly data are only available locally; however, precise calculation of the Fourier transform requires data across the entire 2-D space. Consequently, the obtained G contains a degree of error. Nevertheless, this error can be reduced to acceptable values by using data from a sufficiently large area. In practical calculations, we must replace $F_\mathrm{a}(x, y)$ by $F_\mathrm{a}^*(x, y)$ such that $F_\mathrm{a} = F_\mathrm{a}^*$ when (x, y) lies in a certain region (i.e., Ω) and the possibility exists for $F_\mathrm{a} \neq F_\mathrm{a}^*$ outside Ω. Consider the piezomagnetic field F_p^*, which arises in the magnetization structure corresponding to F_a^*. Because F^* differs from F, the resulting piezomagnetic field F_p^* generally differs from F_p. However, the rapid decay of the magnetic field with increasing distance from the sources means that F_p^* is expected to provide a good approximation of F_p in all areas except those near the margin of Ω. This aspect is briefly demonstrated in the following section.

3. Application to Synthetic Data

To demonstrate the proposed method, the piezomagnetic field is calculated for a situation in which a magnetized prism is embedded in the crust. The concrete configuration of the case study is shown schematically in Fig. 1. The upper and lower bounds of the body are located at depths of 3 and 8 km, respectively, and the N-S and E-W widths of the body are 40 and 60 km, respectively. The inclination and declination of the ambient geomagnetic field are 45° and 0°, respectively, corresponding to mid-latitude regions.

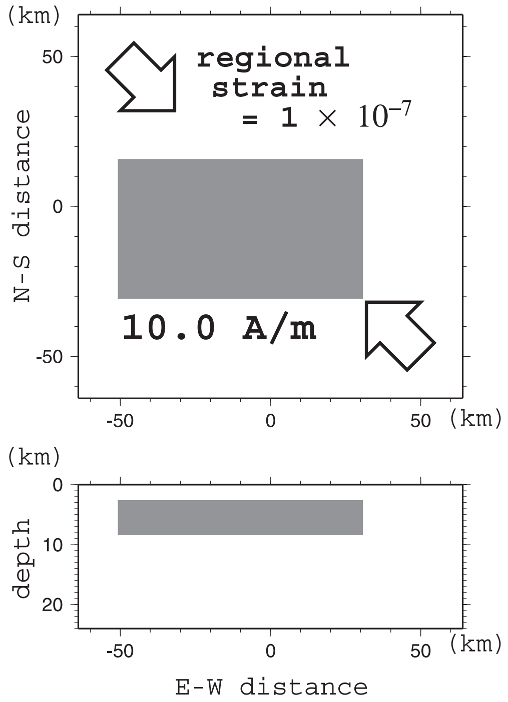

Fig. 1. Configuration of the geological setting for a test of the proposed method in estimating the piezomagnetic field (upper figure: map view; lower figure: cross-section). The rectangle represents the magnetized body (intensity of 10 A/m), and the arrows show the direction of regional strain.

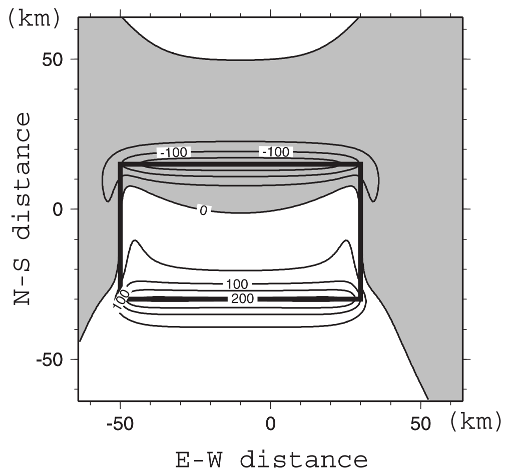

Fig. 2. Anomalies in the geomagnetic total force values generated by the magnetization distribution shown in Fig. 1. Contour intervals are 50 nT. The shaded area indicates negative anomalies.

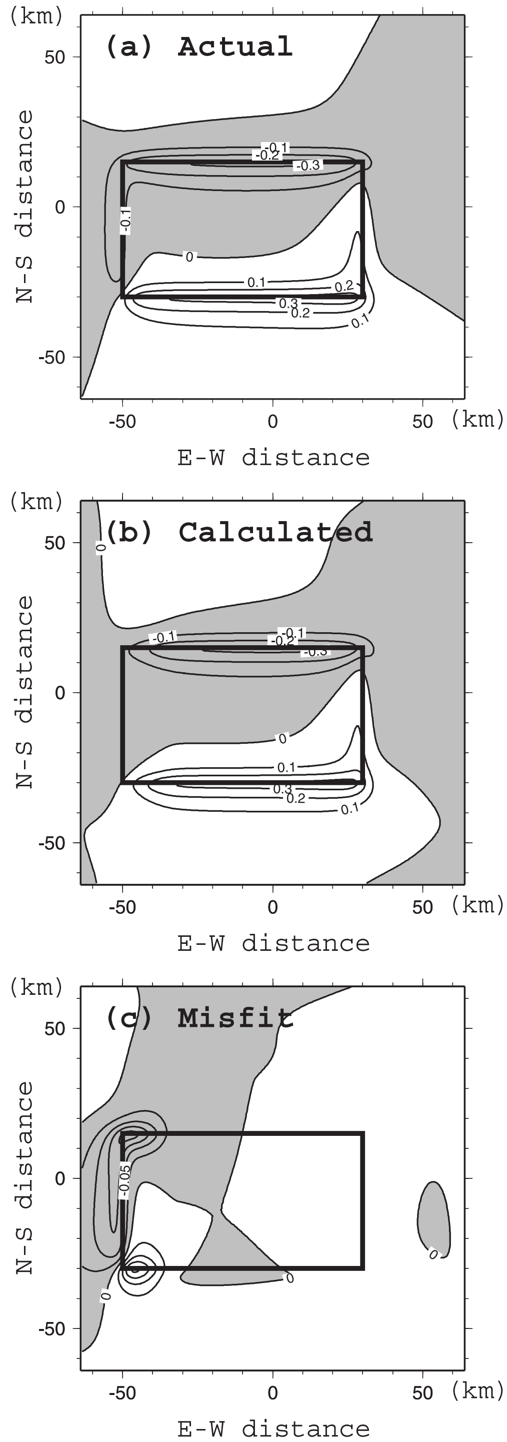

Fig. 3. (a) Actual values of change in the geomagnetic total force field arising from the piezomagnetic field associated with the set-up shown in Fig. 1. (b) Changes due to the piezomagnetic effect, as calculated using the procedure proposed in the present study. (c) Actual minus calculated values. Contour intervals are 0.1 in (a) and (b), and 0.025 nT in (c), respectively. The shaded area indicates negative values.

The regional strain is assumed to be 1.0×10^{-7}, and the rigidity of the crust is 35 GPa. The stress sensitivity is set to 2.0×10^{-2} MPa^{-1}, as indicated by observational studies (e.g., Nishida *et al*., 2004).

The advantage of the above configuration is that it is easy to determine both magnetic anomalies and the actual distribution of the piezomagnetic field using an analytical formula. The magnetic anomaly, ΔF_{a}, is computed using a formula originally given by Bhattacharrya (1964); an equivalent but more useful form has appeared in subsequent papers (e.g., Nakatsuka, 1981; Okubo *et al*., 2005). Figure 2 shows the magnetic anomaly expected to be observed over the magnetization structure given in Fig. 1. The piezomagnetization in the block is calculated according to the constitution law, (1). Because piezomagnetization only appears in the block itself, the piezomagnetic field distribution, ΔF_{p}, is also calculated using the analytical formula (Bhattacharrya, 1964). Figure 3(a) shows the actual piezomagnetic field calculated in this way.

In contrast, the calculation using the proposed procedure is performed as follows. First, magnetic anomalies over a 64×64-km region are calculated at grid points spaced at 0.5-km intervals in each direction, which we use as magnetic anomaly data. The "data" are then 2-D Fourier transformed, yielding the spectrum, ΔG_a. To suppress errors that arise in estimating the 2-D spectrum, we apply the Hanning taper, given by

$$T_{2D}(x, y) = T_{1D}(x)\, T_{1D}(y), \tag{21}$$

where T_{1D} represents the 1-D window function defined by

$$T_{1D}(x) = \begin{cases} 0.5 - 0.5 \cos 2\pi \{(x - L)/L\} & (-L \le x \le -L/2) \\ 1 & (-L/2 < x < L/2) \\ 0.5 - 0.5 \cos 2\pi \{(L - x)/L\} & (L/2 \le x \le L) \end{cases} \tag{22}$$

with L being the half-width of the model region. Subsequently, each spectrum is multiplied by the transfer function $A\left(k_x, k_y; \boldsymbol{n}_a, \boldsymbol{n}_p\right)$, as given in (20). Finally, ΔG_p is converted to ΔF_p by a reverse Fourier transform. Only those spectra with a wavelength greater than 2 km (four times the grid interval) are adopted in constructing ΔF_p. The obtained estimation of the piezomagnetic field is shown in Fig. 2(b).

To assess the validity of values obtained using the proposed procedure, Fig. 3(c) shows the differences between actual and estimated values. Errors as large as 0.1 nT are obtained in a small area near the edge of the calculated region; however, it is natural that such a misfit might arise because information on variations near the edge of the calculation area is lost following application of the taper function, (22). In the central part of the calculated area (i.e., $-L/2 \le x \le L/2$) the misfit is below 0.02 nT, less than 10% of the maximum value of the observed piezomagnetic signals in the region. Therefore, these misfits in the central area are negligible in terms of practical use.

4. Discussion

The proposed method is based on the following three assumptions: the direction of initial magnetization is constant (6), piezomagnetic and magnetic anomalies arising from crustal magnetization are considerably smaller than the ambient geomagnetic field ((7) and (11)), and the regional stress is spatially uniform. The first two of these assumptions do not imply that the proposed method is inferior to the conventional method (e.g., Nishida *et al.*, 2004), as these assumptions are also adopted in other studies. In contrast, the validity of the last assumption (that of uniform regional stress) is questionable when the method is applied to practical situations because the Earth's crust can contain marked spatial gradients in strain rate (e.g., Sagiya *et al.*, 2000). Nevertheless, the piezomagnetic effect is not proportional to strain, but to the stress applied to minerals. Because elastic theory states that traction should be continuous between two media with contrasting rigidities, the assumption of uniform stress is expected to be satisfied, even in some cases in which the strain rate is not uniform. Therefore, the proposed method is applicable to the calculation of piezomagnetic fields for many cases of regional stress.

In practical applications, aeromagnetic data are more easily employed than magnetic anomaly data obtained by ground surveys. When using aeromagnetic data rather than ground data, the downward continuation of data obtained using (17) should be applied in advance. Although downward continuations always result in increased noise in high-frequency components, such noise would be diminished if we employed aeromagnetic data obtained at low altitudes. In Japan, for example, low-altitude and high-density aeromagnetic surveys have been conducted in some areas (e.g., Nakatsuka *et al.*, 2005), and the survey data are available for performing the types of piezomagnetic simulations described in this paper.

Now that we have developed a method to compute the piezomagnetic field due to regional stress, the main difficulty remaining in comparing theoretical and observed values of the piezomagnetic field lies in obtaining observations of the piezomagnetic field. Changes in the magnetic field due to the piezomagnetic effect are less than several nT/yr, even if the stress sensitivity is in the order of 10^{-2} MPa^{-1}; in contrast, changes due to variations in the geomagnetic main field are as large as several tens of nT/yr. Therefore, it is difficult to remove changes in the geomagnetic main field in order to extract piezomagnetic signals, especially in cases in which the site distribution is sparse. Nevertheless, an attempt to extract crustal magnetic fields within the regional geomagnetic field models is currently underway in Japan as part of the Japanese Geomagnetic Reference Field (JGRF) Project (e.g., Ishii *et al.*, 2008). If tectonic signals are precisely extracted, it will be possible to compare simulated and observed values at many points, thereby providing insights into the quantitative nature of the piezomagnetic field.

5. Conclusion

In the case that a uniform regional stress is applied to the heterogeneously magnetized crust, anomalies in the geomagnetic total force field intensities due to the piezomagnetic effect (11) are similar to those due to the magnetization structure (7). This similarity leads a simple relation between the 2-D spectra of these two spatial anomalies (19). Using this relation, we can perform a piezomagnetic simulation without the need to determine the explicit structure of the magnetization intensities within the Earth's crust. The simulation gives rise to minor errors because of inaccuracies in the estimation of the spectrum, although the errors are so small as to be negligible in practical use. Using the procedure, it is possible to compare observed and theoretical values of the piezomagnetic field, thereby helping us to understand the quantitative nature of the piezomagnetic effect.

Acknowledgments. The author thanks Gilda Currenti and Yoichi Sasai for their careful reviews and constructive comments, which led to improvements in an earlier version of the manuscript. The editorial work of the associate editor, Makoto Uyeshima, is also appreciated. James Mori assisted in improving the English in the manuscript. Figures in this paper were drawn using Generic

Mapping Tools (GMT) (Wessel and Smith, 1998).

References

Baranov, V., A new method for interpretation of aeromagnetic maps: Pseudo-gravimetric anomalies, *Geophysics*, **22**, 359–383, 1957.

Bhattacharrya, B. K., Magnetic anomalies due to prism-shaped bodies with arbitrary polarization, *Geophysics*, **29**, 517–531, 1964.

Ishii, Y., T. Toya, and M. Akutagawa, Outline of the first analysis of JGRF data in Kakioka Magnetic Observatory, JMA, *Proc. Conductivity Anomaly Symp. 2007*, 92–97, 2008.

Johnston, M. J. S., R. J. Mueller, and Y. Sasai, Magnetic field observations in the near-field of the 28 June 1992 Mw 7.3 Landers, California, earthquake, *Bull. Seismol. Soc. Am.*, **84**, 792–798, 1994.

Nagata, T., Basic magnetic properties of rocks under mechanical stresses, *Tectonophysics*, **9**, 167–195, 1970.

Nakatsuka, T., Reduction of magnetic anomalies to and from an arbitrary surface, *Butsuri-Tanko (Geophys. Explor.)*, **34**, 332–339, 1981.

Nakatsuka, T., S. Okuma, R. Morijiri, and M. Makino, Compilation of airborne magnetic anomaly maps in Japan from the variety of surveys with long epoch difference, *Proc. 11th IAGA Workshop on Magnetic Obs.*, 230–233, 2005.

Nishida, Y., Y. Sugisaki, K. Takahashi, M. Utsugi, and H. Oshima, Tectonomagnetic study in the eastern part of Hokkaido, NE Japan: Discrepancy between observed and calculated results, *Earth Planets Space*, **56**, 1049–1058, 2004.

Nishida, Y., M. Utsugi, and T. Mogi, Tectonomagnetic study in the eastern part of Hokkaido, NE Japan (II): Magnetic fields related with the 2003 Tokachi-oki earthquake and the 2004 Kushiro-oki earthquake, *Earth Planets Space*, **59**, 1181–1186, 2007.

Okubo, A., Y. Tanaka, M. Utsugi, N. Kitada, H. Shimizu, and T. Matsushima, Magnetization intensity mapping on Unzen Volcano, Japan, determined from high-resolution, low-altitude helicopter-borne aeromagnetic survey, *Earth Planets Space*, **57**, 743–753, 2005.

Oshiman, N., Enhancement of tectonomagnetic change due to non-uniform magnetization in the Earth's crust-two dimensional case studies, *J. Geomag. Geoelectr.*, **42**, 607–619, 1990.

Oshiman, N., M. K. Tuncer, Y. Honkura, S. Baris, O. Yazici, and A. M. Isikara, A strategy of tectonomagnetic observation for monitoring possible precursors to earthquakes in the western part of the North Anatolian Fault Zone, Turkey, *Tectonophysics*, **193**(4), 359–368, 1991.

Sasai, Y., Application of the elasticity theory of dislocations to tectonomagnetic modeling, *Bull. Earthq. Res. Inst., Univ. Tokyo*, **55**, 387–447, 1980.

Sasai, Y., Tectonomagnetic modeling on the basis of the linear piezomagnetic effect, *Bull. Earthq. Res. Inst., Univ. Tokyo*, **66**, 585–722, 1991.

Sasai, Y., Tectonomagnetic modeling based on the piezomagnetism: a review, *Ann. Geofisica*, **44**(2), 361–368, 2001.

Sasai, Y. and Y. Ishikawa, Seismomagnetic models for earthquakes in the eastern part of Izu Peninsula, Central Japan, *Ann. Geofisica*, **40**(2), 463–478, 1997.

Sagiya, T., S. Miyazaki, and T. Tada, Continuous GPS array and present-day crustal deformation of Japan, *Pure Appl. Geophys.*, **157**, 2303–2322, 2000.

Stuart, W. D., P. O. Banks, Y. Sasai, and S-W. Liu, Piezomagnetic field for Parkfield fault model, *J. Geophys. Res.*, **100**, 24101–24110, 1995.

Wessel, P. and W. H. Smith, Improved version of the Generic Mapping Tools released, *Eos Trans. AGU*, **79**, 1998.

K. Yamazaki (e-mail: kenichi@eqh.dpri.kyoto-u.ac.jp)