A computer simulation study on the mode conversion process from slow X-mode to fast X-mode by the tunneling effect

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The mode conversion process from slow X-mode waves to fast X-mode waves by the tunneling effect has been studied by means of computer simulation. On the basis of the new approach by the simulation, we have confirmed that the mode conversion to the fast X-mode waves is especially effective in the case where the width of the evanescent layer which exists between the local UHR frequency and the local fast X-mode cut-off frequency, is of the order of the wave length of the incident slow X-mode waves. In space plasmas, this type of mode conversion process becomes important for the case where the different plasmas form a sharp boundary or a discontinuity at their contact boundary with a scale size of the order of the wave length of the incident slow X-mode waves.

Key words: Mode conversion, numerical experiment.

1. Introduction

The origin of the strong fast X-mode component of planetary radio emission is still controversial in the way that these radio waves are generated. One possible candidate to generate such X-mode radiation is the mode conversion from slow X-mode waves (hereinafter we use the term "slow X-mode waves" to indicate Z mode waves including upper-hybrid mode waves in a frequency range higher than the plasma frequency) by the tunneling effect. Analytical investigations of the tunneling effect have been performed in previous studies (e.g., Budden, 1955; Tang, 1970; Stix, 1992). They analyzed the mode conversion through an evanescent region by using asymptotic solutions of the wave equation and presented the analytical expressions of the efficiency of the mode coupling process. They have also shown this process is effective when the width of the evanescent layer is the order of the wavelength of the incident slow X-mode wave. Tang (1970) investigated a route of this type of mode conversion process analytically and showed the possibility of the indirect generation of fast X-mode waves from slow X-mode waves. In this process, however, a difficulty has been pointed out that, since the waves have to pass through the evanescent region (or layer) between the local upper-hybrid resonance frequency and the local fast X-mode cutoff frequency, the energy conversion rates have been thought to be very small compared with the other indirect processes, such as linear mode conversion to O-mode waves at the plasma frequency (Oya, 1971; Jones, 1977; Budden, 1980) and the nonlinear mode conversion processes (e.g., Roux and Pellat, 1979). Thus, the more direct processes to generate fast X-mode waves has attracted the attention of many researchers such as the cyclotron maser process originally proposed by Wu and Lee (1979).

However, observational evidence suggest the possibility of indirect generation of fast X-mode waves in the space plasmas, such as kilometric radiation accompanied by intense upper-hybrid and Z-mode waves associated with the plasmasphere disturbances in the magnetic storm time (Oya, 1991; Sato, 2001). Therefore, a new simulation tool is required for further investigations to solve the problem of the tunneling effect under various kinds of plasma environments when we understand the physical process of the mode conversion by comparing the theoretical results with the observations in the space plasmas. In the present study, we are trying to develop a new solution tool for the mode conversion process by exactly solving Maxwell's equations and equation of motion of electrons numerically. As for the first step to solve the mode conversion problem by simulation, we have carried out a numerical simulation about the mode conversion process from slow X-mode waves to fast X-mode waves including the finite width effect of the evanescent layer; and evaluated the validity of the simulation by comparing the previous works, first by computing how this type of mode conversion is effective especially depending on the width of the evanescent layer with respect to the wavelength of the incident slow X-mode waves. The situation of the mode conversion process by this tunneling effect is schematically illustrated in Fig. 1. In Section 2, we briefly describe the simulation model and initial parameters used in the present numerical study. We discuss the simulation results in Section 3 and summarize in Section 4.

2. Simulation Model

In our simulation code, we can simulate the propagation of slow X-mode waves in a cold plasma medium under var-

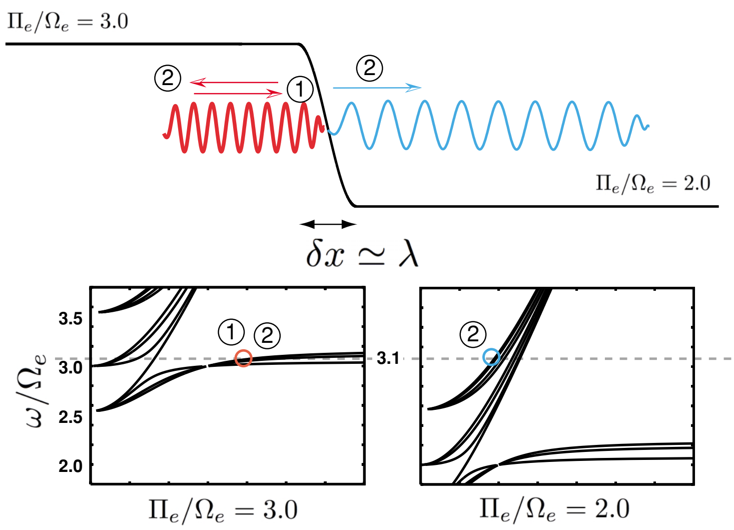

Fig. 1. Schematic illustration of the mode conversion process to generating fast X-mode waves from slow X-mode waves by the tunneling effect. Incident slow X-mode waves generated in the region of $\Pi_e/\Omega_e = 3.0$ propagate into the transition region at time ①, and split into two kinds of waves at time ②; i.e., the reflected slow X-mode waves and the transmitted fast X-mode waves into the region with $\Pi_e/\Omega_e = 2.0$ (see also the corresponding numbers in the dispersion curves in each region). The wave energy can be converted into that of the fast X-mode waves in a form of the transmitted waves.

ious conditions of a density gradient. The background cold electrons are treated as a fluid described by the fluid equation of motion, while the evolution of the electromagnetic field is obtained by Maxwell's equations directly with currents computed from the electron fluid. The basic equations are

$$\frac{\partial \mathbf{v}}{\partial t} = -(\mathbf{v}\cdot\nabla)\,\mathbf{v} + \frac{q}{m}(\mathbf{E} + \mathbf{v}\times\mathbf{B})\,, \tag{1}$$

$$\frac{\partial N}{\partial t} = -\nabla\cdot(N\mathbf{v})\,, \tag{2}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E}, \tag{3}$$

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\mu_0\varepsilon_0}\nabla\times\mathbf{B} - \frac{1}{\varepsilon_0}\mathbf{J}, \tag{4}$$

$$\mathbf{J} = qN\mathbf{v}, \tag{5}$$

where q and m are the charge and the mass of an electron, respectively. In the present model, ions are assumed to be an immobile neutralizing component. To compute the fluid dynamics and the electromagnetic fields numerically, we use a finite difference scheme accurate to the fourth order in space and to the second order in time after Omura and Green (1993). By using our simulation code, we can solve the mode conversion process of various types of plasma conditions and also investigate the nonlinear effects. However, these detailed works are deferred for future studies and as a first step it is important to examine the validity of this new approach for the mode conversion problems by comparing previous works. Thus, we have treated a rather simple case, two-dimensional simulation system on the X-Y plane where the Y-axis is along with the background static magnetic field $\mathbf{B}_0$ and the plasma density gradient is along the X-axis. Although the simulation system is two-dimensional, three-components of velocity and electromagnetic fields are all solved in the Cartesian coordinate system.

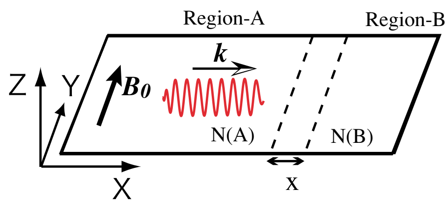

Fig. 2. Schematic illustration of the simulation system.

Table 1. Initial parameters used in the present study.

Grid spacing Δx and Δy	0.06 c/Ω_e
Time step Δt	0.01 Ω_e^{-1}
Number of grids in X-axis L_x	2048
Number of grids in Y-axis L_y	32
Number of time steps	40 000

The simulation system can be divided into three regions along the X-axis as shown in Fig. 2, i.e., region A, region B and the transition region between regions A and B. We assume that cold electrons are uniformly distributed in both regions A and B with different number densities, respectively indicated by $N(A)$ and $N(B)$, while the density difference between regions A and B is smoothly connected in the transition region by using a function form as

$$N(x) = N(A) + \delta N\left(\sin\left(\pi\frac{x}{\delta x}\right) - 1\right)/2, \tag{6}$$

where $x = 0$ corresponds to the center of the transition region, δx is the width of the transition region along with the X-axis, $N(x)$ represents the number density of cold electrons at a position (x, y), and $\delta N = |N(A) - N(B)|$. Note that there is no inhomogeneity along with the Y-axis in this case. To investigate the present mode conversion process, it is important to determine the spatial distribution of the ratio between the plasma frequency Π_e and the cyclotron frequency Ω_e of cold electrons, given by $R = \Pi_e/\Omega_e$. Since we assume a homogeneous background magnetic field, the cyclotron frequency Ω_e is constant, the spatial inhomogeneity of $R(x)$ is therefore only related to the spatial variation of the plasma frequency $\Pi_e(x)$ calculated from $N(x)$. Thus, the spatial scale of the inhomogeneity is characterized by the width of the transition region δx. The boundary conditions are set to be periodic at the Y-boundaries and absorbing boundaries at the X-boundaries.

3. Results and Discussion

In each simulation run, we assume a strictly perpendicular propagating monochromatic slow X-mode wave packet of frequency of $3.1\Omega_e$ in the region A and simulate the propagation of the wave packet into the region B through the transition region. Table 1 gives the initial parameters used in the present study. For the first attempt, we have employed the parameters of the plasmasphere where intense slow X-mode and fast X-mode waves frequently observed in the storm time. In a case where the δx is of the order of the wavelength of slow X-mode waves, it is expected that fast X-mode waves are efficiently generated. Thus, we carried out five simulation runs with different δx, as given by Table 2, while the wavelength λ of the incident slow X-mode wave is $1.29c\Omega_e^{-1}$. By comparing simulation results, we discuss the dependence of the energy conversion rate to the fast X-mode waves as the function of the spatial scale of the density gradient in the transition region. It should be noted that the conversion process reproduced in the present study corresponds to the linear regime, because we assume that the amplitude of wave magnetic field δB of slow X-mode waves is less than $10^{-7}B_0$, where B_0 is the background magnetic field intensity.

Figure 3 shows a wave form of E_x and E_z components of the wave electric field in Run 4 at $Y = 16$. Since we treat the special case where the slow X-mode wave propagates perpendicular to the boundary (also perpendicular to $\mathbf{B}_0$), the simulation results obtained in the present study are symmetric in Y direction. The slow X-mode wave initially generated in the region A propagates toward the transition layer with its group velocity of $v_g = 0.027c$. A part of the energy of the incident slow X-mode waves is converted into the fast X-mode waves, and the rest of the energy is reflected as the backward traveling waves in the form of the slow X-mode waves, satisfying the energy conservation law. This result is quite reasonable because we only included the cold electrons and excluded the finite Larmor radius effects of warm electrons. Since the wave electric field $\mathbf{E_W}$ of fast X-mode mode wave with the frequency of $3.1\Omega_e$ exactly propagating perpendicular to $\mathbf{B}_0$ has a polarization of $\mathbf{E_W} = (0.28, 0, 1)$, when normalized by the Z component of $\mathbf{E_W}$, we can identify that the waves propagating in region B are fast X-mode electromagnetic waves converted from the slow X-mode waves by the tunneling effect through the evanescent region. The conversion rate C given by $C = P_{FX}/P_{SX}$, where P_{FX} and P_{SX}, respectively, denote the Poynting flux of fast X-mode mode and incident

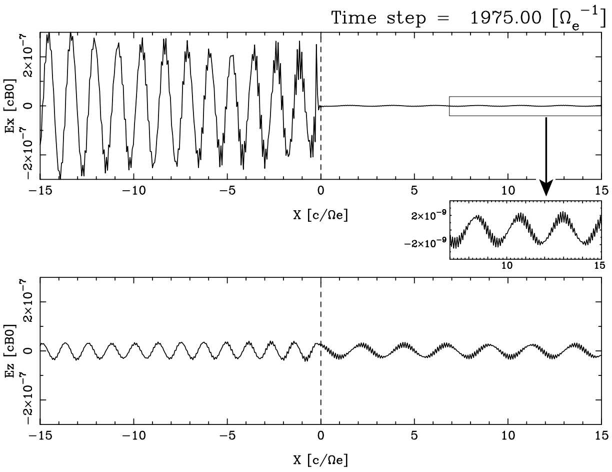

Fig. 3. E_x and E_z component of wave electric field in Run 4. Dashed line shown in each panel represents the position of the center of the density gradient.

slow X-mode mode waves, is estimated to be 0.385 in this case.

As described above, we performed five simulation runs and estimated the conversion rate C depending on the width of the transition region δx. Figure 4 shows the obtained energy conversion rate in each simulation run. We find the energy conversion rate of the tunneling effect varies exponentially with δx. In Fig. 4, we also show the conversion rate derived from the analytical solution obtained by the asymptotic approach, which suggests that the wave power of transmitted wave is expressed by $\exp(-\frac{1}{2}\pi\frac{\delta x}{\lambda})$ (e.g., Stix, 1992). The result indicates that the present numerical results are essentially consistent with the previous results on the basis of the asymptotic approaches and also shows the validity of the present numerical calculations to solve the problem of the tunneling mode conversion processes. We think that the slight difference between these results is caused by the limitation of the approximation in the analytical approaches and also the experimental aspects of the computer simulations. Considering this result, we can confirm that the high energy conversion rates to the fast X-mode mode waves are obtained in a case where δx is of the order of the wave length of initial slow X-mode waves that has been predicted by previous analytical approaches. The simulation results also reveal that at least 1% of the wave power of incident slow X-mode waves can be converted into the fast X-mode mode waves, even in a case where $\delta x \simeq 4\lambda$.

By performing other simulation runs with the different wave frequency of slow X-mode waves, we found that the relation between the conversion efficiency and the width of the evanescent layer normalized by the wavelength of slow X-mode waves is basically the same with Fig. 4. Since the wave spectrum of the slow X-mode waves in the space plasma has been often observed as a banded spectrum from the Z-mode cutoff frequency to upper-hybrid frequency, the wavelengths of slow X-mode waves have various spatial scales according to the dispersion relation. Therefore, as far as the width of the layer is the order of the wavelengths of slow X-mode waves, the efficient mode conversion should

Table 2. The spatial scale of the density gradient used in each simulation run. The wavelength λ of the slow X-mode wave is $1.29c\Omega_e^{-1}$.

	Run 1	Run 2	Run 3	Run 4	Run 5
δx	4 λ	2 λ	1 λ	0.5 λ	0 (sharp boundary)

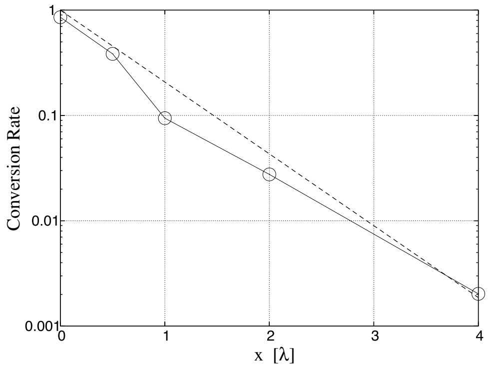

Fig. 4. Energy conversion rates estimated from each simulation run. The conversion rates derived from the analytical solution (e.g., Stix, 1992) is indicated by the dashed line.

be also expected in the case of the banded spectrum of slow X-mode waves.

4. Summary

The generation of fast X-mode mode electromagnetic waves from slow X-mode waves by the mode conversion process, including the tunneling effect, has been investigated by the new approach of the numerical simulation. On the basis of the simulation results, we can clarify that our new approach is useful to investigate this type of mode conversion process and the generation of fast X-mode waves is especially effective in the case where the width of the evanescent (or transition) layer is of the order of the wave length of the incident slow X-mode waves. In other words, the tunneling effect can effectively generate fast X-mode electromagnetic waves in a case where the ratio of Π_e over Ω_e sensitively varies in a spatial scale which is of the order of the wavelength of the slow X-mode waves. In addition, this process obviously requires that the frequency of the originally generated slow X-mode waves in a relatively high density medium is above the cutoff frequency of the fast X-mode waves in the low density surrounding medium. In such cases, this type of mode conversion process becomes important. Such conditions are satisfied in the case of space plasmas where the different plasmas, probably with different origins, form a sharp boundary or discontinuity. This condition might be satisfied in the Earth's auroral region or at the magnetopause, where ambient plasma density is relatively low and the Π_e/Ω_e sensitively varies due to the external perturbations of the plasma density.

In the present study, we treat the simple case that the background plasma as the propagation medium to be a cold plasma medium. By including a thermal plasma component in the simulation model, we can also discuss the effect of the finite Larmor radius effect of the thermal electrons and thus the generation of the electrostatic waves in the transition region where the wave energy will be absorbed by the thermal plasma component, which has been discussed in previous studies on the basis of the asymptotic solutions (e.g., Budden, 1955). In the present work, we conclude that our new tool of computer simulation is quite useful examining the importance of the mode conversion from slow X-mode waves to fast X-mode waves as the source mechanism of strong fast X-mode radiation in the space plasma. We plan to study this mode conversion process for various plasma conditions and also investigate the nonlinear effect in the future studies. The dependences of the energy conversion rates on the incident angle of slow X-mode waves toward the transition region and on the band width effect of the wave spectrum are also left for future studies.

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