Influence of magnetic field variations on measurements by magnetometers using averaging algorithms

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The article is devoted to features of modulus magnetic field measurements by means of proton magnetometers. Inertial characteristics of the magnetometers and dynamic errors of variable magnetic field measurements are discussed. Three basic magnetometers averaging algorithms, processing zero crossing times of free precession, are compared. Theoretical and some numerical estimations of the algorithms work are presented for the linear, square and harmonic variations of magnetic field.

Key words: Proton magnetometer, data processing, absolute measurements.

1. Methodical Problems of Variations Measurement by Means of Inertial Device

The magnetic field variation is continuous function of time. However in practical manner a magnetic field measuring device always has an action time (Ripka, 2001). The magnetometer can not operate instantly. Short as the measurement time will be it is not zero. So a value measured by inertial device is some averaging during a measurement time (Rasson, 1978). There is no problem when magnetic field is constant, but at a presence of variations during measurement time a dynamic error appears. There are tow questions:

1. If magnetic field wasn't constant during measurement, what value must we accept for a fact?

2. What time moment dose this value correspond to?

To take a simple average over measurement time and to correspond it with the center of the measurement time interval is methodical correctly. But averaging algorithms, used at the nuclear precession magnetometers, not give the average of the magnetic field value. The calculation of magnetic field is based on digital processing of the zero crossing times of a free precession signal to estimate an average period of the signal.

Basing on gyromagnetic relation:

$$\nu = \gamma \cdot B, \tag{1}$$

the modulus of magnetic field B is expressed in terms of free precession frequency ν. In general an average magnetic field is not correspond an average period of the signal. Such nonlinear transfer function of magnetometer causes a need for theoretical investigations of different algorithms to answer a question about magnetic field value, which they measure. For three time dependencies of the magnetic variations (linear, square and harmonic) the basic algorithms were considered: Simple Periodometer, Periodometer with Introcycling Treatment and Least Mean Square method (Denisov *et al.*, 1999).

2. Base Formulas

As mentioned earlier, modern proton magnetometers use the zero crossing times to calculate a period of a free precession. Such possibility is based on gyromagnetic relation (1). The period of free precession is inversely to a measured magnetic field (Packard and Varian, 1954).

Figure 1 shows constant magnetic field $\mathbf{B}_0$ and disturbing field $\mathbf{B}_1$, which is varying in time. θ is an angle between the fields. After polarization the magnetization vector becomes free precession on a resultant vector of the magnetic fields with the frequency corresponding to the modulus:

$$B = \sqrt{B_0^2 + B_1^2 + 2B_0 \cdot B_1 \cdot \cos\theta}. \tag{2}$$

Actually the relation of disturbing magnetic field to constant one is too small ($B_1 \sim 1$ nT, $B_0 \sim 50000$ nT). Supposing $B_1/B_0 \ll 1$ one can take Taylor of (2) and take result in the form:

$$B \approx B_0 \left(1 + \cos\theta \cdot B_1/B_0 + 0.5 \cdot (\sin\theta \cdot B_1/B_0)^2\right). \tag{3}$$

The zero crossing times t_i are found from an integral equation, followed from (1):

$$2\pi i = \gamma \int_0^{t_i} B(t)dt. \tag{4}$$

A registration and a record of zero crossing times to a magnetometer buffer enables to employ different digital algorithms to determine the period of free precession and then the modulus of resulting magnetic field. Three digital algorithms will be under consideration (Denisov *et al.*, 1999):

Periodometer –

$$T^P = t_N/N, \tag{5}$$

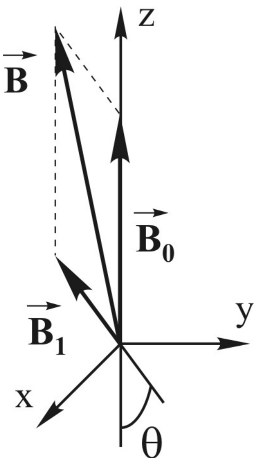

Fig. 1. Resulting magnetic field.

Periodometer with Introcycling Treatmen (PIT) –

$$T^{\mathrm{PIT}} = \frac{1}{(N-[N/3])\,([N/3]+1)} \sum_{i=0}^{[N/3]} (t_{i+N-[N/3]} - t_i), \quad (6)$$

Least Mean Square method (LMS) –

$$T^{\mathrm{LMS}} = \frac{6}{N^3} \sum_{i=0}^{N} (2i - N)\, t_i, \quad (7)$$

where [..] – integer division, N – a number of last recorded time of zero crossing.

Based on the algorithm formulas (4-6), simple Periodometer is expected to give methodical correct value of the magnetic field, as the Periodometer is a simple time averaging. The Periodometer whit Introcycling Treatment and Least Mean Square method are more difficult and special investigation is required. The dynamic properties of the algorithms depend on a character of the magnetic field variations, namely on the time dependence of B_1. General analysis of the algorithms dynamic features is difficult and will be limited by simple time dependencies.

3. Linear Magnetic Field Changing

Let the disturbing field modulus to be line function of the time:

$$B_1(t) = k \cdot t, \quad (8)$$

where k is a speed of the field changing. In that case the integral Eq. (4) is in a form:

$$T_0 i = t_i + (k\cos\theta/B_0)t_i^2/2 + (k\sin\theta/B_0)^2 t_i^3/6. \quad (9)$$

Solving the Eq. (9) for ti by the method of successive approximations supposing $B_1/B_0 \ll 1$, the expression for time zero crossing is:

$$t_i = T_0 i \left(1 - a_1 \cdot T_0 i + a_2 \cdot (T_0 i)^2\right), \quad (10)$$

where $a_1 = (k \cdot \cos\theta/B_0)/2$, $a_2 = (k \cdot \cos\theta/B_0)^2/2 - (k \cdot \sin\theta/B_0)^2/6$ and T_0 is the period of free precession corresponding to B_0.

The expression (10) for the zero crossing times allows us to calculate theoretical value of measured magnetic field for any digital algorithm processing the times (5)–(7). Taking into account gyromagnetic relation between the frequency and the magnetic field (1) the algorithms give the following results:

Periodometer –

$$B^P = B_0\left(1 + T_m a_1 - T_m^2 a_2\right), \quad (11)$$

Periodometer with Introcycling Treatmen –

$$B^{\mathrm{PIT}} = B_0\left(1 + T_m a_1 - 8T_m^2 a_2/9\right), \quad (12)$$

Least Mean Square method –

$$B^{\mathrm{LMS}} = B_0\left(1 + T_m a_1 - 9T_m^2 a_2/10\right), \quad (13)$$

where $T_m = t_N$ is the measurement time.

Analysis of the formulas (11)–(13) shows that at the first order of expansion in terms of small ratio B_1/B_0 all algorithms give the same results. They all measure time average value of the magnetic field and at the case of line magnetic field changing do not have methodical error. The variations along the constant field make the greatest contribution to the measured magnetic field value. At perpendicular orientation of the fields $a_1 = 0$ and the algorithms give different magnetic field values but the difference really is small (for speed of variations 1 nT/s and measurement time 1 s the difference is the order of 10^{-6} nT). This methodical error is out of the magnetometer sensitivity.

4. Square Magnetic Field Changing

For magnetic field changing as parabola

$$B_1 = kt^2, \quad (14)$$

the Eq. (4) takes in a form:

$$T_0 i = t_i + (k\cos\theta/B_0)t_i^3/3 + (k\sin\theta/B_0)^2 t_i^5/10, \quad (15)$$

Expanding the Eq. (15) in a power series of B_1/B_0 and using successive approximations method the solution is:

$$t_i = T_0 i \left(1 - (T_0 i)^2 a_1 + (T_0 i)^4 a_2\right), \quad (16)$$

where $a_1 = (k \cdot \cos\theta/B_0)/3$, $a_2 = (k \cdot \cos\theta/B_0)^2/10 - (k \cdot \sin\theta/B_0)^2/3$.

Now substituting expression for zero crossing times (16) into algorithms formulas (5)–(7) we take correspondingly:

Periodometer –

$$B^P = B_0\left(1 + T_m^2 a_1 - T_m^4 a_2\right), \quad (17)$$

Periodometer with Introcycling Treatmen –

$$B^{\mathrm{PIT}} = B_0\left(1 + 8T_m^2 a_1/9 - 166T_m^4 a_2/243\right), \quad (18)$$

Least Mean Square method –

$$B^{\mathrm{LMS}} = B_0\left(1 + 9T_m^2 a_1/10 - 5T_m^4 a_2/7\right). \quad (19)$$

We can see that at the parabolic changing of magnetic field the algorithms have differences even as in first approximation (Fig. 2). The simple Periodometer gives simple averaging again and the maximal measured value at the parallel orientation of the fields is 0.33 nT. The Periodometer with Introcycling Treatment has transfer constant approximately equal to 0.29 nT and the Least Mean Square method's constant is 0.3 nT.

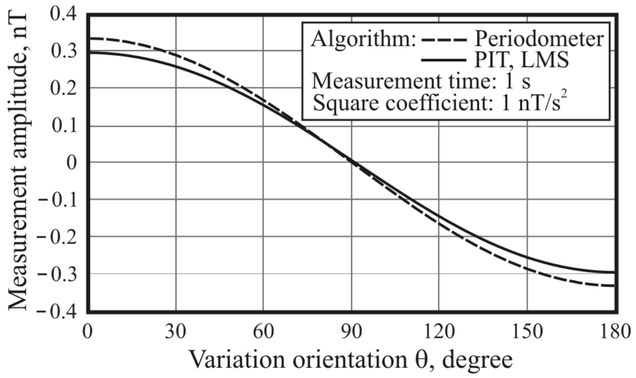

Fig. 2. Angular dependence of the measured field.

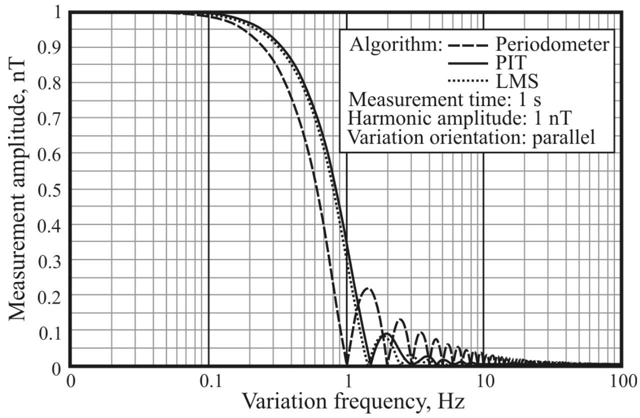

Fig. 3. Transient function of the algorithms.

The difference in measurement results of the algorithms is equal to 10%. For measurement time 1 s and the field changing over the time 1 nT the methodical error approximately is 0.03 nT, it is sizeable for magnetometers with sensitivity up to 0.01 nT.

The comparing of the algorithms allows us to conclude that equal magnetometers with different built in digital algorithms will measure different value of the magnetic field under conditions of square time variations of the field. It depends not on the accuracy of the calculations but on the averaging characters of the calculation methods. However at non-disturbing magnetic situation at the absence of quick variations the difference is not essential.

5. Harmonic Magnetic Field Variations

Suppose disturbing magnetic field changes harmonically:

$$B_1 = k\sin(\Omega t + \varphi), \tag{20}$$

where k is an amplitude, Ω is a frequency and φ is a phase of harmonic.

The Eq. (4) for zero crossing times is more difficult than for previous tow cases:

$$T_0 i = t_i + \frac{2k\cos\theta}{B_0\Omega}\sin(\Omega t_i/2)\sin(\Omega t_i/2+\varphi) + \left(\frac{k\sin\theta}{2B_0}\right)^2\left[t_i - \frac{1}{\Omega}\sin(\Omega t_i)\cos(\Omega t_i + 2\varphi)\right]. \tag{21}$$

Solving the Eq. (21), the zero crossing times are expected as:

$$t_i = T_0 i(1-a_2) - \frac{4a_1}{\Omega}\sin(x_i/2)\sin(x_i/2+\varphi)\left[1-\frac{4a_1}{\Omega}\sin(x_i+\varphi)\right] + \frac{a_2}{\Omega}\sin(x_i)\cos(x_i+2\varphi), \tag{22}$$

where $a_1 = (k\cdot\cos\theta/B_0)/2$, $a_2 = (k\cdot\sin\theta/B_0)^2/4$, $x_i = \Omega T_0 i$.

Putting (22) into algorithms formulas we are taking the measured magnetic field values under conditions of linear approximation:

Periodometer –

$$B^P = B_0\left[1 + 4\frac{a_1}{x_N}\sin(x_N/2)\sin(x_N/2+\varphi)\right], \tag{23}$$

Periodometer with Introcycling Treatmen –

$$B^{\mathrm{PIT}} = B_0\left[1 + 36\frac{a_1}{x_N^2}\sin(x_N/3) \times \sin(x_N/6)\sin(x_N/2+\varphi)\right], \tag{24}$$

Least Mean Square method –

$$B^{\mathrm{LMS}} = B_0\left[1 - 24\frac{a_1}{x_N^2}\left(\cos(x_N/2) - \frac{2}{x_N}\sin(x_N/2)\right) \times \sin(x_N/2+\varphi)\right], \tag{25}$$

where $x_N = \Omega T_m$.

The digital algorithms work like a filter, suppressing and passing frequencies selectively. The transient function of the algorithms depends not only on the frequency of disturbing field harmonic but on its phase too. On Fig. 3 the transient functions of the digital processing methods are presented and the phase of input harmonic is chosen to maximize the gain.

In the first, the essential decay of harmonic amplitude caused by the magnetometers inertial characteristics is observed since as low frequencies as $0.1/T_m$.

In the second, all algorithms pass low frequencies up to $1/T_m$ and suppress high frequencies. The transfer functions have a quasiperiodic character. The frequencies divisible by K/T_m are suppressed, where coefficient K is different for each algorithm. It is clear what for simple Periodometer $K = 1$, because the average for sinusoid exactly packing in the measurement time is null. For PIT $K = 1.5$, for LMS method $K \cong 1.43$.

Thus there are situations for some harmonics when identical magnetometers with different built-in algorithms will measure different values. For example for sinusoidal magnetic field of $0.5/T_m$ frequency and 1 nT amplitude, Periodometer shows 0.65 nT amplitude, PIT gives 0.8 nT, LMS gives 0.78 nT (Fig. 3). The methodical error at the presence of sinusoid variation of 1 nT amounts to 0.15 nT. It is exceeds the magnetometer sensitivity.

Table 1. Time dependence of coefficient C under 50 Hz disturbances.

τ_m	−0.5 s	−1 s	−1.5 s	−2 s
C^P	1.3×10^{-2}	6.4×10^{-3}	4.2×10^{-3}	3.2×10^{-3}
C^{PIT}	7.3×10^{-4}	1.8×10^{-4}	8.1×10^{-5}	4.6×10^{-5}
C^{LMS}	4.9×10^{-4}	1.2×10^{-4}	5.4×10^{-5}	3.04×10^{-5}

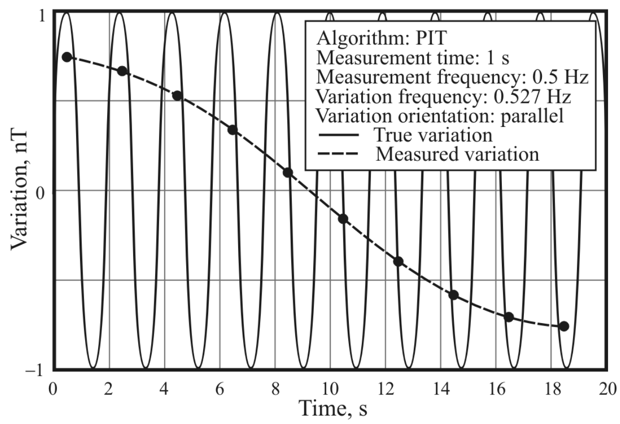

Fig. 4. The effect of frequency substitution.

It should be mentioned that processing algorithm causes not only amplitude distortions in to harmonic but it causes the frequency substitution of one. The effect of frequency substitution shown on Fig. 4 is explained by Nyquist sampling theorem (Nyquist, 1928).

It is possible to use filtration features of the algorithms for noise control. For example, to exclude magnetic disturbances induced by power-line noise of 50 Hz and 60 Hz for measurement by simple Periodometer, it is necessary to set a magnetometer measurement time divisible by 100 ms, for PIT by 150 ms, for LMS by 143 ms. In this case amplitudes 50 Hz and 60 Hz harmonics are suppressed. In other cases at short measurement times the harmonics can give the significant contribution. For frequencies much greater than $1/T_m$ the approximation formula for a maximal contribution of harmonics is

$$\Delta B \approx k \cdot C, \qquad (26)$$

where $C^P = 2/(\tau_m \Omega)$ for simple Periodometer, $C^{\mathrm{PIT}} = 18/(\tau_m \Omega)^2$ for PIT, $C^{\mathrm{LMS}} = 12/(\tau_m \Omega)^2$ for LMS and τ_m is the nearest time to T_m at which the disturbances influence is maximal. Numerical estimations of the factor C are represented in the Table 1.

Thus there is an essential influence of variable fields of $50 \div 60$ Hz, especially for fast measurements (1 s cycle time, 0.5 s measurement time), which are perspective for modern observatories. It is interesting, that for the update signal processing algorithms the influence of the industrial disturbances is less than for simple Periodometer.

6. Conclusions

The different integrating characteristics of the algorithms can leads to methodical and dynamic errors at the measuring magnetic variations. Under normal observatory conditions the errors are not essential, on the contrary at a magnetic storm or at a presence of industrial disturbances the errors are sizeable. The nonlinear magnetic field variations lead to great errors, essentially depending on the type of processing algorithm.

In spite of that fact that the simple Periodometer is methodically correct algorithm, it has great dynamic error in comparing with the modern algorithms such as Periodometer with Introcycling Treatment and Least Mean Square method. Besides that at the short measurement times the $50 \div 60$ Hz disturbances have a greater effect on a Periodometer result than on modern algorithms one. The ways of algorithmic noise control by means of a choice of measurement time are proposed.

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