Gravitational Lens Effect and Measurement of Stellar Mass

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Abstract. An application of the gravitational lens effect is proposed for measuring the mass of a single star. The configuration of a foreground star (deflector), a background star (source) and the observer is considered. When configuration shift occurs along the direction of source-deflector separation seen from observer, the observed angular shift is suppressed by the gravitational deflection shift. On the contrary, when the configuration shift is orthogonal to that direction, the angular shift is enhanced by the deflection itself. Using these effects, not only the distance of the deflector, but also its mass can be measured. If a deflector is in the distance within 100 pc, with the mass $\sim 10 M_{\odot}$, and the source-deflector separation is about 1 arc second, 1 AU configuration shift will cause 10 μas deflection shift. VLBI technology will enable us to detect this angular shift in the very near future. It is also discussed that the mass of a deflector can be determined only through detecting the shift of the separation magnitudes in the case that there exist multiple sources.

VLBI is one of the highest resolution observation method in astronomy. Recently its resolution runs up to 50 μas. This high resolution will enable us to measure the stellar masses by detecting gravitational lens effects.

As far as gravitational lens effects are concerned, much attention has been paid for luminosity changing effect, that is called microlensing (REFSDAL, 1964, 1966; LIEBES, 1964; PACZYNSKI, 1986). Recently, some candidate events of this effect have been observed (AUBOURG *et al.*, 1993; ALCOCK *et al.*, 1993; UDALSKI, *et al.*, 1993). Now we consider the other effect of gravitational lens, i.e. the gravitational deflection of radio wave. Here we would like to make a short and intuitive description. For more complete description on this matter, see Ref. (HOSOKAWA *et al.*, 1993).

Radio wave from a background star (source, S) to observer (O) is deflected by the gravitational field of a foreground star (point mass deflector, P). Configuration of S, P and O and the coordinates taken here are illustrated in Fig. 1. The relation between true separation angle β and apparent separation angle θ is represented as (SCHNEIDER *et al.*, 1992)

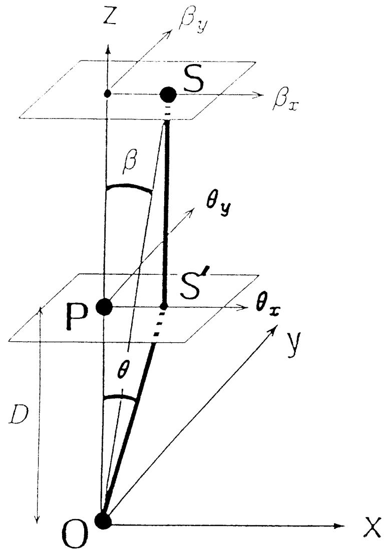

Fig. 1. Reference configuration of gravitational lensing. A point mass deflector P is at a distance D from the observer O at the origin of the coordinates and P is on the z axis. A background source S on the x-z plane has an angular separation β from the deflector. A light emitted by S passes through S′ and reach O because of the gravitational deflection by P. The actually observed angle θ is the angle ∠POS′.

$$\theta = \beta + \frac{4GM}{c^2 D\theta}, \tag{1}$$

where M is the mass of the deflector, G is the gravitational constant and D is the distance between observer and deflector. The true angle β cannot be observed, so the deflection angle $4GM/c^2D\theta$ neither. However, if the observer shifts its position by $r = (x, y)$ on x-y plane, the shift of the apparent separation can be detected. The components of this shift is calculated as follows.

$$\Delta\theta_x = \frac{x}{D}\left(1 - \frac{4GM}{c^2 D\theta^2}\right), \quad \Delta\theta_y = \frac{y}{D}\left(1 + \frac{4GM}{c^2 D\theta^2}\right). \tag{2}$$

The difference between x/D, y/D and $\Delta\theta_x$, $\Delta\theta_y$ is illustrated in Fig. 2. If we suppose $D \sim 100$ pc, $\theta \sim 1$ arc second, $M \sim 10M_\odot$, and $x, y \sim 1$ AU that can be easily obtained by the Earth revolution, then

$$\frac{4GMx}{c^2 D^2 \theta^2} \sim 10^{-5} \text{ arc second.} \tag{3}$$

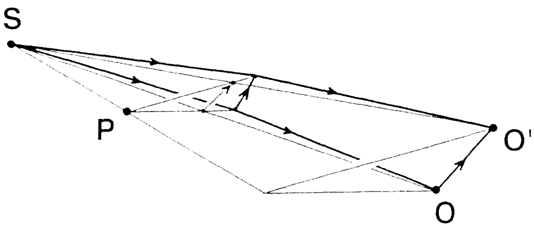

Fig. 2. Difference between true shift and apparent shift. Deflection angle varies if the impact parameter changes with observer's position shift from O to O'.

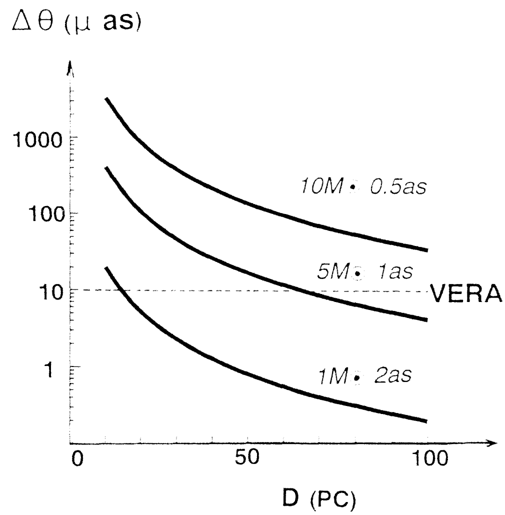

Fig. 3. Relation between distance of P and deflection shift. Three cases are shown according to the mass of deflector and separation between deflector and source.

This angular shift will be detectable in some VLBI project that are now in proceeding (LESTRADE *et al.*, 1992; SASAO *et al.*, 1993; JONES *et al.*, 1993). For example, Japanese VERA project is aiming this resolution. In that time, mass and distance of the deflector can be obtained simultaneously.

$$D = \frac{1}{2}\left(\frac{x}{\Delta\theta_x} + \frac{y}{\Delta\theta_y}\right), \quad M = \frac{c^2\theta^2}{8G}\left(\frac{x}{\Delta\theta_x} - \frac{y}{\Delta\theta_y}\right). \tag{4}$$

Figure 3 will show us the relation between distance of the deflector and the difference of deflection angle. When the resolution becomes higher, the deflection by farther and lighter deflectors can be detected.

For now, the practical way to obtain the long baseline for observer's position shift is to make use of the orbital motion of the Earth. In this case, after the proper motion of deflector is removed, the apparent trajectory of the source seen from observer on the θ_x-θ_y plane will make a parallactic elipse. But that elipse will be distorted by the gravitational deflection effect. This distortion depends on P's ecliptic latitude and the direction of PS, and the same order of deflection difference as shown in Fig. 3 will be observed. A simple case that S is on the meridian of P is illustrated in Fig. 4.

In the case of actual observation, the decomposition of the angular shift into two components x, y with required accuracy might be difficult. In that case, if multiple sources are well aligned with a deflector, such decomposition is not necessary and only through the variation of the magnitude of the separation angles, M and D can be obtained. The result on general case is rather complicated. For simplicity, here we take a special case. Suppose two sources S_1 and S_2 are located as shown in Fig. 5. The direction of the observer's shift is denoted by ϕ. Because of the deflection by the deflector P, according to the observer's shift the apparent separation angle between S_1 and S_2, denoted by θ_{12} will change, while true separation should not change. The angular shift of θ_{12} is,

$$\Delta\theta_{12} = -\frac{8GMr}{c^2D^2\theta^2}\cos\left(\phi - \frac{\pi}{4}\right). \tag{5}$$

This observable quantity is proportional to the mass of the deflector and its ϕ dependence is not so sensitive. The expected value for this quantity is the same order as in the case considered before.

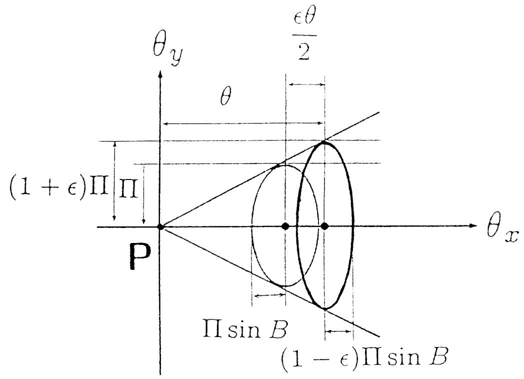

Fig. 4. Distortion of parallactic ellipse: Case $\varphi = 0$. Π is the annual parallax. Compared to the ellipse in the absence of gravitational lensing, the semi-minor axis becomes shorter and the semi-major axis becomes longer by the amount $\varepsilon = 4GM/c^2D\theta^2$ whose typical value ~0.001.

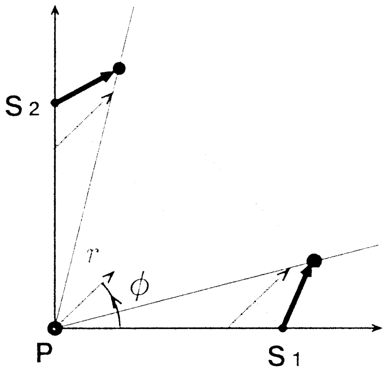

Fig. 5. Case of two sources. The view from the observer is shown. The deflector P is at the origin. With observer's position shift *r*, S shift their apparent position as shown by solid arrows. Dotted arrows show true angular shifts.

To summarize, high resolution narrow-field astrometry will enable us to determine the mass of a nearby single star. When the distance of a star is less than 100 pc and some appropriate sources are within 1 arc second in its background, not only the distance but also the mass of the star can be determined by detecting the variation of angular separation between the star and sources. For this purpose, the resolution higher than 10 μas might be required. VLBI is one of the most hopeful technology to achieve such resolution in the near future.

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