Modelling Study of the Cometary Ly $\alpha$
Brightness from a Time-varying H$_2$O Source

Osamu Ashihara

University of Industrial Technology, Sagamihara 229, Japan

1. Introduction

The first suggestion of the existence of a huge hydrogen cloud around comets dates back to Biermann (1968), and was actually observed in comet Tago-Sato-Kosaka by Code et al. (1970). Since then, observations in the Ly $\alpha$ wavelength region have been followed to the present for comets such as Benett, Kohoutek, Kobayashi-Berger-Milon, West, and several other fainter comets.

Parallel to these observations, clouds have also been studied from a variety of theoretical viewpoints: What species of parent molecules are responsible for the production of atomic hydrogens? How they are disintegrated with distance from the nucleus? What kind of forces are working upon the released hydrogen atoms and how are they distributed in the surrounding space? To what degree the orbital motion of a comet is exerting an influence over the outermost region of the cloud? (the atoms lying in such a distant region were produced in earlier times when the heliocentric distances of the comet and the activities were quite different from those at present) Depending on the aspects studied, and on the region of interest in the cloud, different methods have been so far developed.

Among the numerous discoveries brought about by the recent space missions for Comet Halley, one of the most unexpected is the quite variable photometric features of the cloud. The Japanese spacecraft Suisei reported, already in its early stage of observations, that the brightness of the Ly
α intensity of Comet Halley is significantly changing from time to time (Kaneda et al., 1986a; 1986b; 1986c), suggesting the existence of active sites on the cometary nucleus surface and their complicated interrelation with the nucleus rotation (or precession). This prompted us to set on a work that can well simulate the time-variable nature. In this paper, we report the modelling work based on which the early (particularly in 1985) UV data taken by Suisei were analyzed.

All the conventional models of hydrogen clouds (Haser, 1957; Keller and Thomas, 1975; Keller and Meier, 1976; Festou, 1981), assume that the parent molecules for atomic hydrogens are isotropically released from a point source. This assumption is followed in this work despite the above mentioned fact on the nucleus surface which suggests an initially strong directionality of the relevant physical quantities (jet). This is due to: (a) that the successive collisions and photodissociations (isotropic ejection of atomic hydrogens*), are thought, after all, to produce a circumstance where the significance of the initial directionality of the released parent molecules is greatly reduced, and (b) that the spatial extent of our concern is so vast that a unisotropic distribution of the parent molecules in the near-nucleus region is unlikely to give a substantial effect on the overall distribution of atomic hydrogens. In other words, a spherical symmetry of all relevant physical quantities were assumed for the present work. Hence an effect of the radiation pressure, one of the interesting problems, cannot be incorporated into the present formalism (readers, who are interested in this aspect, may refer to Finson (1968), Keller and Thomas (1975), Keller and Meier (1976), Kitamura et al. (1985), and Ashihara (1986)). Anyway, we put emphasis on examining the time dependent features of the problem.

2. Formulation

Here we derive the equation which governs the density distribution of the atomic hydrogens yielded from photodissociations of OH and H₂O, the latter being ejected from a point source with a time varying production

*Strictly speaking, in the photodissociations of H₂O and OH, the angular distribution of the ejected H atoms is not isotropic. For instance, in the case of H₂O, it is known both theoretically and experimentally, that the distribution of the fragmented H atoms obeys a law, 1 + βP₂(cosθ) (Greene and Zare, 1982; Segev and Shapiro, 1982; Shapiro and Bersohn, 1982; Andresen et al., 1984), where P₂ is the Legendre function of order 2 and θ is a scattering angle of the ejected atom measured from the direction of the
rate $Q(t)$. The notations used are as follows: $v_{H_2O}$ is the radial (constant) velocity of H$_2$O ejected from the point source and $v_{OH}$ is that of OH. In the following, the both outflow velocities are assumed to be equal and simply written as $v = v_{H_2O} = v_{OH}$. Furthermore, $\tau_{H_2O}$ and $\tau_{OH}$ are the photodissociation lifetimes, respectively, for H$_2$O and OH, which depend only on the heliocentric distances of the comet, and $\tau_H$ is the lifetime of H against photoionization and/or charge transfer with the solar wind particles. Now, let $n_{H_2O}(t, r)$ and $n_{OH}(t, r)$ be the densities of H$_2$O and OH at a time $t$ and a distance $r$. Then we have

$$n_{H_2O}(t, r) = \frac{Q(t - r/v)}{4\pi r^2 v} \exp(-r/\tau_{H_2O} \cdot v)$$

(1)

and

$$n_{OH}(t, r) = \frac{Q(t - r/v)}{4\pi r^2 v} \frac{\tau_{OH}}{\tau_{H_2O} - \tau_{OH}} \cdot \{\exp(-r/\tau_{H_2O} \cdot v) - \exp(-r/\tau_{OH} \cdot v)\}.$$ 

(2)

It is readily confirmed that they satisfy the usual continuity equations for H$_2$O and OH having a term $\partial/\partial t$. It is also seen that they are a revision of the classical expression by Hasinger (1957), in which the constant production rate $Q$ was simply substituted by $Q(t - r/v)$. Since it is known that photodissociations of H$_2$O and OH due to the solar UV yield hydrogen atoms consisting of several velocity components of a rather restricted range (cf., Keller (1976): also see Dixon (1970) and Van Dishoeck and Dalgarno (1984)), we simply assume that they are each produced in their central velocities. We express, in the photodissociations of H$_2$O, the channel index, the quantum yield, and the velocity of the ejected atomic hydrogens, by $i$, $\omega_i$, and $v_i$, respectively, and in those of OH, by $j$, $\omega_j$, and $v_j$, respectively.

The electric field vector of incident light. The influence on the present problem, which not only degrades a symmetry but necessitates a polarization formulation in the radiative transfer and as a result considerably increases computational complexities, remains so far unstudied although it is quite an interesting topic.
\[(\sum_i \omega_i = \sum_j \omega_j = 1)\]. Then the density of the atomic hydrogens resulting from the photodissociations of H\(_2\)O and OH will be expressed, respectively, as

\[
n^i_{H(\text{H}_2\text{O})}(t, R) = \int_0^{\infty} 2\pi r^2 dr \int_0^{\pi} \sin \theta d\theta \omega_i \frac{n_{\text{H}_2\text{O}}(t - \rho/v_i, r) \exp(-\rho/v_i\tau_{\text{H}})}{4\pi \tau_{\text{H}_2\text{O}} v_i} \frac{\exp(-\rho/v_i\tau_{\text{H}})}{\rho^2} \tag{3}
\]

and

\[
n^j_{H(\text{OH})}(t, R) = \int_0^{\infty} 2\pi r^2 dr \int_0^{\pi} \sin \theta d\theta \omega_j \frac{n_{\text{OH}}(t - \rho/v_j, r) \exp(-\rho/v_j\tau_{\text{H}})}{4\pi \tau_{\text{OH}} v_j} \frac{\exp(-\rho/v_j\tau_{\text{H}})}{\rho^2}, \tag{4}
\]

where \(\rho = (r^2 + R^2 - 2rR \cos \theta)^{1/2}\) and \(R\) is the distance from the nucleus. The static (time-independent) version of these expressions is that of the so-called vectorial model (Festou, 1981).

Substituting Eqs.(1) and (2) into Eqs.(3) and (4), we obtain (using \(2\rho dp = 2rR \sin \theta d\theta\))

\[
n_i(t, R) = \frac{\omega_i}{8\pi v_i l_{\text{H}_2\text{O}}} \int_0^{\infty} dr \frac{\exp(-r/l_{\text{H}_2\text{O}})}{r} \int_{|r - R|}^{r + R} d\rho \frac{\exp(-\rho/l_i)}{\rho} \cdot Q(t - \rho/v_i - r/v) \tag{5}
\]

and
\[ n_j(t, R) = \frac{\omega_j}{8\pi v_j l_{OH}} \frac{\tau_{OH}}{R (\tau_{H_2O} - \tau_{OH})} \int_0^\infty dr \frac{\exp(-r/l_{H_2O}) - \exp(-r/l_{OH})}{r} \cdot \int_{|r-R|}^{r+R} d\rho \frac{\exp(-\rho/l_j)}{\rho} Q(t - \rho/v_j - r/v) \] (6)

where \( l_{H_2O} = v \cdot \tau_{H_2O} \), \( l_{OH} = v \cdot \tau_{OH} \), and \( l_i = v_i \cdot \tau_H \). For further progress we have to specify the functional form of \( Q(t) \). We assume in the present work that it is given by step function, i.e.,

\[ Q(t) = Q_k (\text{const}) \quad \text{for} \quad t_{k+1} < t < t_k. \] (7)

It is noted that this retains a broad possibility of the functional representation for \( Q(t) \), as is fulfilled by making the divisions \( t_k \) smaller.

Let the quantities \( a_i^k \) and \( b_i^k \) be defined as \( v_i(t - t_k - r/v) \) and \( v_i(t - t_{k+1} - r/v) \), respectively. Then after some manipulations, we obtain

\[ n_i(t, R) = \frac{\omega_i}{8\pi v_i l_{H_2O} R} \sum_{k=0}^{\infty} Q_k \int_0^\infty dr \frac{\exp(-r/l_{H_2O})}{r} \cdot \left[ E \left( \frac{-\min(r+R, b_i^k)}{l_i} \right) - E \left( \frac{-\max(|r-R|, a_i^k)}{l_i} \right) \right] \] (8)

and
\[ n_j(t, R) = \frac{\omega_j}{8\pi v_j l_{OH}} \frac{\tau_{OH}}{\tau_{H_2O} - \tau_{OH}} \cdot \sum_{k=0}^{\infty} Q_k \int_0^\infty dr \frac{\exp(-r/l_{H_2O}) - \exp(-r/l_{OH})}{r} \cdot \left[ E \left( \frac{-\min(r + R, b_j^k)}{l_j} \right) - E \left( \frac{-\max(|r - R|, a_j^k)}{l_j} \right) \right], \]  

where \( E(-x) \) is the exponential integral function, defined by

\[ -\int_x^\infty \frac{e^{-t}}{t} dt. \]

Here the integrand must not be negative and, hence, \( r \)'s interval allowed in the integration is classified into three cases. If we introduce the variables as

\[ r_r = (v_i(t - t_{k+1}) - R)/(v_i/v) - 1 \]

and

\[ r_l = (v_i(t - t_k) - R)/(v_i/v) + 1, \]

they become

a) for \( R > v(t - t_{k+1}) \)

\[ \max(r_l, 0) < r < \max(r_r, 0), \]
b) for \( v(t - t_{k+1}) > R > v(t - t_k) \)

\[
\text{max}(r_l, 0) < r < r_r, \quad (14)
\]

c) for \( v(t - t_k) > R \)

\[
r_l < r < r_r. \quad (15)
\]

Once these densities are calculated one can easily obtain the column density along a given line of sight, which points in a direction deviating to an angle Θ from the observer-cometary nucleus line, as

\[
N(t, a, Θ) = \int_{-a \cos Θ}^{\infty} \left( \sum_i n_i(t, R) + \sum_j n_j(t, R) \right) dz, \quad (16)
\]

where \( a \) is the distance between the observer and comet and \( R = (z^2 + (a \sin Θ)^2)^{1/2} \). Based upon this formula we made a variety of test runs by changing the parameters such as \( t_k \) and \( Q_k \) so as to obtain the best fittings for the Ly α brightness distributions observed by *Suisei* from Comet Halley.

3. Numerical examples

We show below some of the numerical examples calculated from the formula (see Eq.(16)). For a demonstrative purpose, we take the form of \( Q(t) \) simply as a stepwise constant function, as shown in Fig. 1, and repeat ten times the cycle (consisting of active and inactive phases). All other quantities, to describe a geometrical configuration among the sun, comet and observer, are those actually registered from Halley’s comet, the sun and the spacecraft *Suisei* at the epochs. The absolute values of \( Q(t) \) were given such that its long-term average equals the value recommended by Divine *et al.*
Fig. 1. The occurrence diagram of outburst events. Ejection of H$_2$O molecules from a point source is schematically drawn. They were repeated 10 times with a period 53 hours and a duration time half the period.

(1986). For simplicity we further assume that the photodissociation channels of H$_2$O and OH are unique for each, the former yielding the hydrogens at 20 km/s and the latter, those at 8 km/s.

The epochs tentatively computed are, one, at Nov. 1, 1985, when the comet is still at 1.92 AU apart from the sun and the photodissociation lifetimes of H$_2$O and OH are fairly large, and the other, at Feb. 9, 1986, when the comet is just at perihelion (0.59 AU) and the lifetimes are much shorter. Figures 2 and 4 show the spatial intensity distributions of the Ly $\alpha$ calculated for the above two epochs, while Figs. 3 and 5 show the time-dependent features of the intensity at a fixed line of sight (They are $10^4$ km and $10^5$ km projected distances from the nucleus, in Figs. 3i and 3ii, respectively, and $10^4$ km and $10^6$ km, in Figs. 5i and 5ii, respectively). In the figures, curve (A) is the result due to the present work and curve (C) is that due to the Haser's model, which is a static model and only shown for reference. Curve (B), of which the explanation is not given in the text, is the result obtained by assuming that H$_2$O and OH, the parents of H, reside at the origin (point source) after the release from the nucleus and thus atomic hydrogens are also yielded at the origin. This assumption leads to a great simplification of the final expression and yet gives a fairly satisfactory
approximation to case (a), except for the region near the nucleus.

Curves (4) and (5) show the contributions of 20 km/s and 8 km/s hydrogen components to the profile (b), respectively, and the curves labelled 1, 2, 3, …, the accumulated contributions of the periodic outburst occurrences to the profile (b): the curve labelled 1 considers only the most recent event, and the one labelled 2 the two most recent events, and so on. The gradual access to curve (b) with the repetition is readily discerned.

We first discuss the spatial intensity profiles of the Ly α (Figs. 2 and 4). It is generally seen that (c) < (a) < (b) in the inner region of the coma, but (b) < (a) < (c) in the middle transient region and (b) ≈ (a) < (c) in the outer coma. With decreasing heliocentric distances of the comet, the differences existing among the three models (a, b, c) become rapidly diminishing (particularly so between (a) and (b)). The non-negligible differences between (c) and (a, b) exist only at the most interior part of the coma, where the Haser’s model largely underestimates the actual hydrogen density. In those profiles of spatial intensity distribution, it is seen that the influence of time-varying production is readily smoothed out with the repetition of outburst
Fig. 3. The Ly $\alpha$ intensity variations at a fixed line of sight (los) along the Suisei trajectory. The epoch is Nov. 1, 1985 UT. The projected distances of the fixed los are, respectively, $10^6$ km for Fig. 3i and $10^5$ km for Fig. 3ii.
events, leaving only a trace of its finiteness on the slight undulation of the curve (Fig. 4). Very roughly summarizing, it can be said that the differences among these models are, at any rate, not so striking. Quite a contrasting situation, however, arises if we see the intensity profiles, taking time as an independent variable at a certain fixed line of sight.

In any of Figs. 3i, 3ii, 5i, and 5ii, the form of $Q(t)$ (Fig. 1) is exactly reflected on the rises and falls of the curves, though with a due shift dependent on the projected distance of the line of sight of our current concern. The amplitude of the undulation is seen to decrease with increasing projected distance. This is the natural outcome, from the fact that, for a given line of sight the closer it is to the nucleus, the more abundantly the atomic hydrogens of recent production are lying relative to those of earlier production. Thus we can conclude that, if one is to determine the $Q(t)$ profile as accurate as possible (under the assumption of an isotropic point source), then it is desirable to choose a concerned line of sight as close as possible to the nucleus. This may, however, cause a rather complicated situation to solve a radiative transfer problem for the Ly $\alpha$ line, because under the current circumstances the central portion of the Ly $\alpha$ wavelength region
Fig. 5. The Ly α intensity variations at a fixed los along the Suisei trajectory. The epoch is Feb. 9, 1986UT. The projected distances of the fixed los are, respectively, $10^4$ km for Fig. 5i and $10^6$ km for Fig. 5ii.
would no more be optically thin for intensities higher than about 20 » 30 kR.

References


**Appendix**

The conversion of the column density of the atomic hydrogens along a line of sight to the observed Ly $\alpha$ intensity is made as follows. We assume that the cloud dimension of our interest is still small when compared with the distance between the sun and comet and also with that between the comet and observer. Then the observed intensity $I$ of the resonantly scattered radiation will be,

$$I = N \frac{\pi e^2}{m_e c} f F_\odot p(\chi) / r_H^2,$$  \hspace{1cm} (A1)
where \( f \) is the oscillator strength of the Ly \( \alpha \) transition (Allen, 1955), \( F_\odot \) the photon flux of the solar Ly \( \alpha \) at 1 AU: the empirical formula of the flux values \( w.r.t. \) \( F_{10.7} \) index can be seen, \( e.g., \) in Vidal-Madjar (1975) and Bossy \( et \) \( al. \) (1981). \( r_H \) is the distance (AU) between the sun and comet and \( p(\chi) \) the phase function of the resonant Ly \( \alpha \) radiation, where \( \chi \) is an angle spanned by the sun, comet and observer. The explicit form of the normalized phase function is given by (Brand and Chamberlain (1959))

\[
p(\chi) = \frac{1}{4\pi} \left( 1 + \frac{1}{4} \left( \cos^2 \chi - \frac{1}{3} \right) \right).
\]

(A2)

If the quantities are expressed in the units of \( 1/cm^3 \) for \( N \), in photons/(cm\(^2\) \cdot s \cdot \) A for \( F_\odot \), and \( (\pi e^2/m_e c) f \) is given a value \( 5.448 \times 10^{-15} \), then \( I/(10^6/4\pi) \) gives an intensity in the so-called Rayleigh unit (1R = \( 10^6/4\pi \) photons/(cm\(^2\) \cdot s \cdot str)).