Simulation Studies on the Formation Processes of the Saturnian Ringlets

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1. Introduction

As has been summarized in the second paper of this chapter, the rings are popular existence for all four giant planets. The rings have, however, their own different characteristics reflecting the condition of each giant planetary system possessing significant records of the past evolution periods of the giant planetary system. Therefore, it is important to clarify the mechanism sustaining the ring systems.

The most apparent and well developed ring system is the Saturnian system. For this ring system the studies have long been continued to understand the mechanism to form fine structure of the distribution of the rings. After the historical studies of rings by Galileo-Galilei, Huygens, Laplace, and Roche (see Helden (1984) and references in it), Alexander (1962) has pointed out that the stable rings can only exist when they are made of many small bodies. Thus, basic understanding that the rings consist of small bodies formed in the region of Roche’s limit has already been established. That is, tidal forces always disturb the accretion processes of the small bodies to grow into larger bodies.

In the early phase of the studies the concept of the “Jet Stream” has
been proposed by Alfvén and Arrhenius (1976). The idea of the Jet Stream is expressed by the following processes:

a) The ionized particles rotate with the rotating magnetic field.

b) When the particles grow the size by attachments, the charged particles are eventually neutralized, and these particles start to take the Keplerian orbits.

c) At each moment when the neutralized particles make crossing the region of dense concentration of material forming the rings, the particles reduce their original angular momentum until the particles make orbits of the rings and the elastic collisions make no significant effect.

The analytical studies by using the Boltzmann equation have been made relating to the collection of the ring elements by Baxter and Thomson (1971; 1973). The studies on the dynamics of rings have made growth starting from the research of collisional Keplerian system (Trulsen, 1972a; 1972b; 1972c). In these studies it has been thought that inelastic collisions are source of the collection of the ring material by reducing the distribution of original angular momentum to the groups of the rings elements. Their results show that negative diffusion takes place owing to the inelastic collision.

Two kinds of effects can be considered for the gravity effects between element bodies of the ring; the first is the effect that appears in the case of the close encounters and the second is the effects of mutual interactions of the many bodies. The studies of Hämnen-Anttila (1983) have been done for the first effects and the studies of Salo and Lukkarri (1982) have been carried out for the second case; the results show that the ring system is much more stabilized by these effects of the gravity. All these results indicate that the possible accumulation of the element particles can take place within a limited range; but these can not interpret the fine structure of the ringlet that have been observed by Voyager spacecraft.

For understanding the formation processes of the ring, computer simulation is one of the important basic approaches. Trulsen (1972a) has made computer simulation, using his $\beta$ model where three dimensional trace of the particles has been done under the conditions of the inelastic collision for complete spheric model without any destruction of each particle in the collision processes. The inelastic collision under the study is therefore change only the momentum and the energy. The rate of the inelasticity is given to
Fig. 1. Ringlets in the Saturnian ring system (Presented by NASA, P-23953). The narrowest ringlets in this diagram corresponds to the total width of the present simulation segment 360km.
be $\beta = 2$ for perfect elastic collision and $\beta = 1$ for perfect inelastic collision. His results show that the ring system becomes thinner when $\beta$ comes close to 1. Brahic and Hénon (1977) have also made computer simulation; their results also basically support the results of Trulsen (1972a).

In the present works, then, computer simulations for understanding the formation processes of ringlets (see Fig. 1) have been carried out for the many-body systems under the Keplerian motion with mutual gravitational interaction. The stages of the computer simulations are divided into two regime; the first is the regime of the microscopic phase and the second is the phase of the ringlet instabilities. The first stage is considered for the case where the distribution of real size of the element bodies of the ring makes inhomogeneous concentrations yielding seeds of the ringlets. The second stage is the growth phase of the ringlets increasing the density modulation of the distribution of the element bodies. In this work we have concentrated ourselves to the second stage of the simulation. For the second stage, super particle scheme has been employed; in this treatment, the particles with radius of 1km are considered to represent the characteristic size of the density modulation in the distribution of the element bodies of the ring. The results show that the density waves generated in the ring media are origin of the ringlets which have been observed by Voyager in the Saturnian ring system; i.e., we can understand the ringlet formation as the result of the controlling effects due to the self gravity between element bodies.

2. Model and Methods of Computer Simulation

Region and Basic Equations

A small section in the ring is selected as a system for simulation, that is rotating surrounding main bodies to satisfy the Keplerian orbit conditions. The simulation then made for the evolution of the perturbational change of the positions from the initially given places. The size of the system is selected as given in Figs. 2a and 2b, that is, the thin square with 360km length and 44km width is set in the midst of the ring which is located at $R = 1 \times 10^5$ km from the center of Saturn and is rotating with angular speed of $1.99 \times 10^{-4}$ rad/s keeping the Keplerian orbit. Within this elementary box, 363 or 990 of super particles are distributed. The particles are governed by the equations
Fig. 2. The region selected for the present computer simulations; (a) for the case of 363 super-particles and (b) for the case of the 990 super-particles.

\[ \ddot{r}_i = \nabla_i (U_i + U_i^*), \]  
\[ U_i = G \left( \frac{M_p}{r_i} + \frac{m_i}{r_i} \right), \]  
and
\[ U_i^* = \sum_j' G m_j \left( \frac{1}{\Delta_{ij}} - \frac{r_i \cdot r_j}{r_j^3} \right), \]

where \( r_i \) is the position vector of \( i \)-th particle and \( \Delta_{ij} = | r_i - r_j | \). Furthermore, \( M_p \) and \( m_i \) are the masses of the central planet and \( i \)-th particle, respectively. Corresponding quantities are given in Fig. 3.

**Super Particle Model**

The super particles treated in the present simulations, consist of sphere with radius of 1km. Because the sphere represents the group of the real
element bodies made with ice in the ring, the density $\rho$ of the super particles is given by

$$\rho = N(r_e/r_s)^3$$  \hspace{1cm} (4)

where $N$, $r_e$, and $r_s$ are the number of element bodies within a given super particle, radius of element bodies, and the radius of the super particles, respectively. When super particle consists of element bodies that completely fill inside of the superparticles, $\rho \simeq 0.5$; and when element particles are aligned with distance of 5m (for $r_e = 1$m), $\rho \simeq 0.3$. In the present simulation, the case of $\rho = 0.1$ is selected; this value corresponds to the case where element particles are aligned every 3.4m.

Collisions between super particles can not be expressed by the simple $\beta$-model. Even if the super-particles are overlapping each other this does not necessarily mean the direct collision; we should allow penetration effects
of the super particles. Then a critical zone is defined with size $a_{in}$ for the limit where the super-particles start to interact each other (see Fig. 4).

When the centers of two super particles approach within the limit of $a_{in}$, the interaction takes place by changing velocity and angular momentum, *i.e.*, velocities both in the direction of tangential vector $n$ and the normal vector $k$ (see Fig. 5) and the angular frequency of the rotation $\omega_i$ can be expressed as

![Diagram](image)

**Fig. 4.** Model for superposition of the super-particles; $a_{in}$ is defined as the diameter of limit region to make direct collision with the other super-particle.

![Diagram](image)

**Fig. 5.** Geometrical feature and corresponding vectors at the moment of the collision of the super-particles. $k$ and $n$ denote the normal vector in parallel to the collisional motion, and tangential vector respectively and $V_i$ and $V_j$ are velocity vectors of $i$-th particle and $j$-th particle, respectively, just before the collision. The super particles $i$ and $j$ are making rotation with angular frequency $\omega_i$ and $\omega_j$ respectively also just before the collision.
\[ V'_{n_i} = \frac{1}{2} \left[ V_{n_i} + V_{n_j} + \frac{I}{a_{in}^2 m - I} \left\{ \left( a_{in} - \frac{a_{in} \mu}{1 + \Delta V_{kk} \gamma} \right) (\omega_i + \omega_j) + \left( \frac{a_{in}^2 m}{I} - \frac{\mu}{1 + \delta V_{kk} \gamma} \right) (V_{n_i} - V_{n_j}) \right\} \right] , \]

\[ V'_{k_i} = \frac{1 - \sqrt{1 - \epsilon}}{2} V_{k_i} + \frac{1 + \sqrt{1 - \epsilon}}{2} V_{k_j} , \]

and

\[ \omega'_{i} = \omega_{i} - \omega_{j} + \frac{1}{I - a_{in}^2 m} \left\{ a_{in} \left( 1 + \frac{\mu}{1 + \Delta V_{kk} \gamma} \right) (V_{ni} - V_{nj}) + \left( I - \frac{a_{in}^2 m \mu}{1 + \Delta V_{kk} \gamma} \right) (\omega_i + \omega_j) \right\} . \]

where

\[ \Delta V_{kk} = | V_{k_i} - V_{k_j} | \]

In the above, \( V_n \) and \( V_k \) are, respectively, the tangential and normal components of the velocity of a particle, and additional subscripts \( i \) and \( j \) denote quantities of \( i \)-th and \( j \)-th particles. Furthermore, \( m \) and \( I \) are the mass of the super particles and the moment of inertia around its mass center, respectively, and \( \epsilon \) is the ratio of elasticity of collision. The velocity and angular velocity of particles after interaction are denoted with \( (\cdot)' \). Using \( \mu \) and \( \gamma \), we can express the change of rotational velocity during the interaction of two super particles: \( i.e. \), relative velocity \( \Delta V_b \) of the interacting boundary (see Fig. 6) changes between the moments before and after the interactions as

\[ \Delta V_{ba} = \frac{\mu}{1 + \Delta V_{kk} \gamma} \Delta V_{bb} , \]

where \( \Delta V_{ba} \) and \( \Delta V_{bb} \) denote the relative velocity of the boundary in after-interaction case, and before-interaction case, respectively; \( \Delta V_{kk} \) is the normal component of relative velocity at the moment of the collision of the two super particles (see Fig. 6). Equation (9) can be rewritten as
\[ \Delta V_{ba} = \mu \Delta V_{bb} - \Delta V_{kk}(\Delta V_{ba} \gamma). \]  

(10)

Therefore, \( \mu \) is a simple ratio of the deceleration of the relative boundary velocity while \( \gamma \) is complex nonlinear term that basically contribute to transfer the relative velocity of the parallel movement into the rotational velocity after the collision.

**Initial Conditions and Parameters**

Simulations have been carried out for two cases of the number \( N_\gamma \) of the particles, \textit{i.e.,} \( N_T = 363 \) and 990. Within the section of 360km \( \times \) 44km, the particle distributions are set to gather in the middle part of the section for the case of \( N_T = 363 \) and to fill all of the section for the case of \( N_T = 990 \) (see Figs. 2a and 2b). At the boundary, the cyclic boundary conditions are employed; \textit{i.e.,} the particles which make encounter with a boundary appear with the same velocities from the opposite side of the boundary. The initial condition of the velocity is set to have the Keplerian velocity with the angular velocity of

\[ \omega_{r_i} = \sqrt{GM_{\text{Saturn}}/r_i^3} \]  

(11)
and with $V_r = 0$ for the radial direction; that is, all of the simulation runs have started from “cool” condition. The important factor of the evolution of system is, therefore, completely depending on the self-gravity of the ring system. In Table I, 12 cases of the computer-runs are given with labels for corresponding combinations of the parameters. The labels in this table will be cited in the following section instead of describing detailed values of the parameters.

3. Results of the Simulations

The Case of No-superposition

The case, of $a_{in} = 2000m$, means that the super particles are not allowed to make any super-position. Though this case is too strongly interacting compared with the actual case, we can use the result to see the tendency of the contributing parameters. That is, results of the $\epsilon$-dependence can be seen by a comparison between cases of A1aX and A3aX, as given in Fig.
7. In this figure, the results of the distribution of the super particles are given for the time $t = 21T_0$ and $t = 20T_0$, respectively, for $\epsilon = 0.99$ and $\epsilon = 0.5$ cases, where $T_0$ denotes the rotation period of the super particles around Saturn. The results show that, for the case of large inelasticity, the distribution moves into the state indicating the formation of ringlets.

In Fig. 8, the results for the combination of parameters A1aX and A1aZ are compared to see the effects of the $\gamma$ of considering particles, which is considered to affect to the equivalent elasticity for interval of $T_0$ starting from $t = 21T_0$ for A1aX and $t = 20T_0$ for A1aZ. The case of $\gamma = 1$ cor-
Fig. 8. Time evolution of the distribution of the super-particles for one rotation period (a) for the parameter set of A1aX (see Table I) starting from $21T_0$ and (b) for the parameter set of A1aZ (see Table I) starting from $20T_0$ ($T_0$ denotes one rotation period around the main planet). In both panels each segment of the diagram corresponds to the time passage of $(1/8)T_0$. 
Fig. 9. Plots of simulation results for N=990, case with $\epsilon = 0.99$, $\mu = 1.0$, $\gamma = 0.1$ for $a_{\text{in}}$ = 2km. The color code corresponds to: blue for 0.001, green for 0.001 $\sim$ 0.01km/s, yellow for 0.01 $\sim$ 0.1km/s, red for 0.1km/s $\sim$ 1km/s and orange for 1 $\sim$ 10km/s. The top panel (a) shows the distribution at 16.9$T_0$ and panel (b) shows the distribution at 17.0$T_0$, (see Fig. 7 for $T_0$).
responds to the rough interaction of the super particles giving rotation to the other super particles while the case \( \gamma = 0 \) means smooth interaction without transport of the parallel momentum to the angular momentum. The results show that the collection of the superparticles takes place in the radial direction when the momentum of the super-particle is transported from parallel to the rotation at the time of the collision.

The distribution of the velocity has also been studied as has been given in Fig. 9 where the simulation results are plotted for the \( N = 999 \) model with velocity denoted by color codes. It is interesting that not only the particle makes ringlet distribution but they also show systematically inhomogeneous distribution of the velocities.

Though we can grasp basic processes of the controlling effects of \( \epsilon \) and \( \gamma \) for interaction of the super-particles, in the case of non-overlapping, these results are over emphasized in the collision effects compared with actual case because super particles are not real particles but defined as collected regions of the element bodies of the ring. We should, therefore, consider the cases where the super-position of the super particles are allowed.

The Case of the Superposition

1) \( a_{in} \)-dependence

When we select the \( a_{in} \) value to be smaller than 2000m, the case corresponds to the simulation where the super particles are allowed to make overlapping each other. The rate of overlapping therefore depends on the value of \( a_{in} \).

In Figs. 10a to 12b, the distributions of the super-particles are given for different values of the parameter \( a_{in} \): \( i.e., \ a_{in} = 852, 632, 424, \) and \( 200, \) respectively, in Fig. 10, 11, 12a, and 12b. In Figs. 10 and 11, a- and b-panels indicate the same results; \( i.e., \) panels (a) in Figs. 10 and 11 show the distribution pattern and panels (b) show the number of particles within each bin that is made by dividing \( 360 \)km range into \( 360 \) bins. Results given in these figures show that the fine structure of the ringlet is formed for the large values of \( a_{in} \) while there appears a tendency to take a large simple ring form when the interaction becomes weak (\( i.e., \) for the case of small \( a_{in} \)).

2) \( \epsilon \)- and \( \mu \)-dependence

In Figs. 13a and 13b, the results for the cases of C2bX and C1aX are given, respectively, for \( (\epsilon = 0.99; \ \mu = 1.0) \) and \( (\epsilon = 0.71; \ \mu = 0.5) \). It is
Fig. 10. Time evolution of the distribution of the super-particles for one rotation period $T_0$. (a) The distribution is directly indicated starting from $t = (14 + 1/8)T_0$. (b) The distribution is plotted as histograms for 360 bins which are defined by dividing 360km range by 1km unit. The time evolution is indicated every $T_0/8$, from $t = 8T_0$ to $t = 11T_0$. 
Fig. 11. The same as Fig. 10 for both panels (a) and (b), but for the case of C1aX (see Table I).
Fig. 12. The same as Fig. 8, but for the cases of D1aX (a) and E1aX (b).
Fig. 13. The simulation results to compare the effects of different inelasticity given by the parameters for the parameter set (a) C2bX and (b) C1aX; the data are displayed in the same format with panel (b) of Fig. 10.
apparent that for highly inelastic interaction (the case of C2bX) the distribution of ring material makes a large simple structure while the fine ringlet structures are revealed for relatively low elasticity (the case of C1aX).

4. Discussions

Ringlet formation

The present simulation results show that ringlets with characteristic lengths of gaps ranging from 20 to 50km are formed in radial directions. From the analyses of the simulation results, the processes for formations of the ringlets can be understood as the results of ensemble of the effects of the self-gravity waves with the absorption of the momentum due to relative velocity by inelasticity of the collision processes. The characteristic size of the ringlet therefore depends on the inelasticity and density of the distributing element bodies in the rings.

From the observations of ISS (Image Science; Smith et al., 1981; 1982), PPS (Photopolarimetry; Lane et al., 1982), UVS (Ultraviolet Spectroscopy; Holberg et al., 1982) and RSS (Radio Science; Marouf et al., 1986; 1983) on board Voyager spacecraft the fine structures of the ringlets have been observed. In Fig. 14, one of the results from the B ring for this observations (Lane et al., 1982) is indicated. When we pick up a result from our simulation for C1aX, (see Fig. 15), we can identify that the same categories of density modulations are produced. This evidence suggests that we can understand the formation mechanism of the ringlets by the same physics that is clarified by the present simulation works.

![Graph](image_url)

Fig. 14. The data showing the density fluctuation in the region of the wave train in the inner B ring (after Lane et al., 1982).
Fig. 15. Average accumulation profile of the ring material obtained by the present studies on the simulation of the ringlets. The total numbers of the super-particles within the range of 44km in the azimuthal direction are plotted for each bin obtained by dividing 360km range (for 99,820km to 100,180km measured from the center of the planet) with 1km width.

Synchronized Modulations

As have been indicated in the results of simulations given in Figs. 10b, 11b, and 13a, positions of formed ringlets make oscillation coinciding with the rotation period $T_0$ of orbital motion of the element bodies. This means that the stable groups of the distributed element bodies take slightly elliptical orbits surrounding the main planet, Saturn.

When we define the vector $\vec{r}$ and the eccentricity $e$ as given in Fig. 16, it follows that

Fig. 16. Schematic orbit of the element body surrounding the central planet.
\[ e = 1 + 2 \left( \frac{r V_\theta}{G M_{\text{Sat}}} \right) \left\{ \frac{V_r^2 + V_\theta^2}{2} - \frac{G M_{\text{Sat}}}{r} \right\} \]  
(12)

and

\[ r = \frac{1}{1 - e^2} \frac{(r^2 V_\theta)^2}{G M_{\text{Sat}}} \]  
(13)

where \( r, V_r, \) and \( V_\theta \) denote the distance of the super-particles from the center of Saturn, radial velocities, and azimuthal velocities of the super-particles, respectively; \( G \) and \( M_{\text{Sat}} \) are the gravity constant and the mass of Saturn.

In Figs. 17a to 17d, the plots of the eccentricity \( e \) versus the position

Fig. 17. Plots of the eccentricity of the distributed super-particles in the particles in the ringlets for four parameter sets (a) C1aX, (b) C1aW, (c) C1bX, and (d) C2aX, (see Table I).
of the simulation regions are given with the histogram of the distribution of the super-particles. Tendency of the concentration of these super-particles becomes apparent, i.e., when the distribution of the element bodies shows ringlets formation apparently they belong to groups that take same eccentricity.

5. Conclusion

The computer simulations for super-particles which consists of the sphere with radius of 1km with density $\rho = 0.1$ have been made for the Keplerian system with 44km width in the azimuthal direction and 360km length in radial direction centered at the distance of $10^5$km from the center of Saturn. The superparticles represent the group of the element bodies of the rings which possibly consists of small ice material with radius of 1m in average. The simulation has been carried out for the cyclic boundary conditions starting from the cool initial condition where the evolution of the system is traced starting from zero speed in the rest frame of the super particles that makes rotation with the Keplerian motion with complete circular orbits.

Considering nature of the super-particles, the condition of the encounters of each super-particle are carefully treated; i.e., the encounter takes place with overlapping. The parameter to express the overlapping is given by $a_{in}$ that is defined as the diameter of the region of the limit where the super particles are allowed to make overlapping. The interaction is also taking place under the condition of the inelastic collision where the momentum of the relative motion of the super particle is absorbed by random oscillation of the element bodies during encounter; the inelasticity is expressed by $\epsilon$. The frictional rotation of the super-particles is also considered by introducing the parameter $\mu$ and $\gamma$.

The simulation results show that ringlet formation is made as the results of the internal gravity field that is formed as ensemble of the distributed ring materials. The ordered motion of the super-particle then makes density modulation with characteristic length between 10 to 50km. When the collision processes take place under the highly elastic condition, the mutual motion shows the scattering of the distribution, rather than to form the collection into the ringlets. For formation of the ringlets, therefore, inelastic
conditions for the collision processes are also important.

The results of present simulation provide similar data for the ringlets structure with those observed by Voyager. We can conclude then, that the selfgravity effects with inelastic collision are basic causes for formation of the ringlets.

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