Concepts and Modelling in Geomorphology: International Perspectives,
Eds. I. S. Evans, R. Dikau, E. Tokunaga, H. Ohmori and M. Hirano, pp. 101–114.

Green's Function of Mass Transport and the Landform Equation

Masashige HIRANO

Graduate School of Science, Osaka City University,
Sugimoto 3-3, Sumiyoshi-ku, Osaka 558-8585, Japan
e-mail: hirano-h@kcc.zaq.ne.jp

Abstract. Transport of the earth surface material associated with diffusion brings a landform change, and the resulted sediment or debris thickness is approximated by a normal distribution. The essential feature of this geomorphic process is described in terms of the transport distance and the spatial spreading of the masses. A unit mass on a slope may move at a rate b downward and spread at a diffusivity a, simultaneously. Thus, we may introduce a kind of Green's function which gives the diffusion over the area at and the transportation over the distance bt of the unit mass situated on a given point at $t = 0$ during the time t. The Green's function is applied to any kind of mass movement, though the constants a and b may change by the processes. Topographic change due to the masses distributed continuously in the source area is derived by integration of the Green's function of mass transport, and the result gives the general solution of the landform equation which means that the rate of erosion is proportional to the convexity and gradient of land surface. The solution derived from the landform equation shows the retreat and subduing of a vertical scarp at $x = 0$ with time in the simplest case. Application of the Green's function to such a present day mass movement as the Ontake landslide in 1984 is also possible, though several problems are open to the future investigations including that from the statistical point of view.

Keywords: Mass Transport, Normal Distribution, Green's Function, Landform Change, Landform Equation, Ontake Landslide

GREEN'S FUNCTION FOR MASS TRANSPORT

Green's function in potential theory defines the effect of a point source to the surrounding area under a definite boundary condition. Thus, the concept of Green's function in broader sense is useful even for such an irreversible process as the heat conduction or mass diffusion as stated by Carslaw and Jaeger (1959). Considering a region with a particular condition on its boundary, we can introduce a Green's function for landform change corresponding to a unit mass of earth surface material at a particular point.

Transport of the earth surface material associated with diffusion brings a topographic change. The essential feature of this geomorphic process is described in terms of the transport distance and the area spreading of mass on slopes by the process concerned. Therefore, it is possible to state that the resulted distribution of debris is approximated at a first step by a normal distribution with the mean μ and the standard deviation σ given by

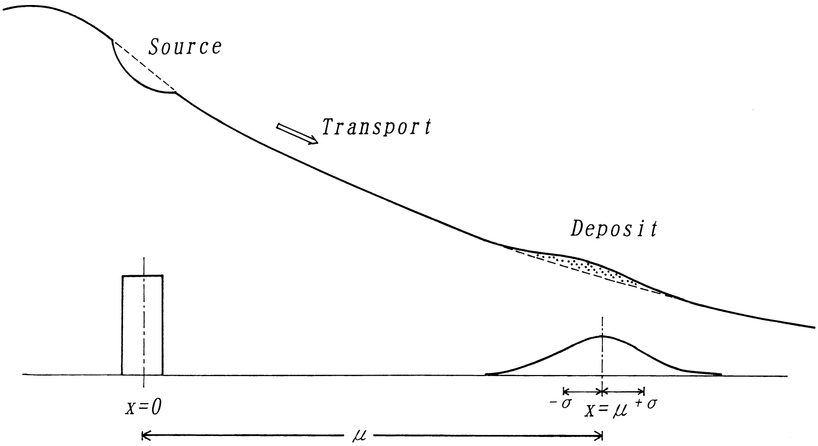

Fig. 1. Schematic presentation of fundamental feature of mass movement in terms of the transportation and diffusion of surface material on a slope. Travel distance is given by $\mu = bt$, and the spreading (diffusion) by $\sigma = \sqrt{2at}$.

$$N(\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{(x-\mu)^2}{2\sigma^2} \right\} \tag{1}$$

as shown by Fig. 1 with the origin $x = 0$ at the source. It is also noted here for further discussion on this account that the normal distribution is the approximation to the binomial distribution which does correspond to the debris distribution over a finite spatial range.

A unit mass on a slope moves at a rate b downward and spreads at a diffusivity a, simultaneously over the time t. Thus, we may introduce

$$u_* = \frac{1}{2\sqrt{a\pi t}} \exp\left\{ -\frac{(x-bt-\xi)^2}{4at} \right\} \tag{2}$$

as a kind of Green's function for a unit mass as a unit instantaneous source situated at $x = \xi$ at $t = 0$ in one dimensional case with the boundary at infinity. We may put $\xi = 0$ in Eq. (2) for the comparison with Eq. (1) or Fig. 1, and this means that the source in Eq. (1) is at $x = 0$ as shown in Fig. 1. The function given by Eq. (2) defines the diffusion over the area at and the transportation over the distance bt during the time t. The constants a and b may change by the geomorphic processes.

The scheme shown in Fig. 1 concerns the mass movement chiefly, but it is possible to apply this even to the sediment transport based on its essential feature.

Thus, it is reasonable to consider that Eq. (2) concerns the mass transport in broader sense and is applied to various geomorphic processes including the landslide or sediment transport.

In general case, the mass distributes continuously along x-axis from $x = -\infty$ to $x = +\infty$ forming a slope, and the initial distribution can be expressed by $f(x)$ at $t = 0$. The function $f(x)$ may define the land surface elevation as the initial condition. Topographic change for the continuous masses is thus given by integration of Eq. (2), and we have

$$u(x,t) = \frac{1}{2\sqrt{a\pi t}} \int_{-\infty}^{+\infty} f(\xi) \exp\left\{ -\frac{(x - bt - \xi)^2}{4at} \right\} d\xi \tag{3}$$

Equation (3) describes the redistribution of the continuous masses associated with the transportation and diffusion on the slopes, and the process brings topographic change.

It should be emphasized here additionally that the movement (transportation) is restricted to down-hill orientation on land surface, and this is an important difference of geomorphic mass transport from the usual diffusion. Derivation of the mass from a source (erosion) and its later settlement (deposition) are both directly connected with the change of land surface elevation.

LANDFORM EQUATION

The next problem is to know the differential equation which has the Green's function given by Eq. (2). By substitution of $b = 0$ and putting $u_* = h_*$, Eq. (2) reduces to

$$h_* = \frac{1}{2\sqrt{a\pi t}} \exp\left\{ -\frac{(x - \xi)^2}{4at} \right\} \tag{2'}$$

which gives the effect of a point source without transportation, and this is the chief portion of Green's function for the equation of diffusion or heat conduction given by

$$\frac{\partial h}{\partial t} = a \frac{\partial^2 h}{\partial x^2} \tag{4}$$

for one-dimensional case in an infinite media (Carslaw and Jaeger, 1959). Equation (4) was first introduced by Culling (1960) to analyse the erosional process of hill slopes.

Introducing a new variable u given by

$$h = u \exp\{\alpha t + \beta x\} \text{ with } \alpha = \frac{b^2}{4a} \text{ and } \beta = \frac{-b}{2a}$$

Eq. (4) for h changes by this transformation to

$$\frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial x^2} - b \frac{\partial u}{\partial x} \tag{5}$$

for which Eq. (2) is the Green's function. In other words, Eq. (2) is the solution of Eq. (5) corresponding to a unit instantaneous source on $x = \xi$ at $t = 0$. It is noted here that $\partial u/\partial x$ is positive for positive b as the surface elevation decreases with time by erosion usually.

Equation (5) is the landform equation proposed by Hirano (1966) in order to describe the landform change in one dimensional case and studied systematically (Hirano, 1968, 1975, 1976). The equation means geomorphologically that the rate of landform change concerns the slope gradient and curvature with the elevation u. Equation (5) is possible to be explained as a process response model (Hirano, 1990), though weathering of bed rock is still open as a future problem.

Equation (3) gives the general solution of Eq. (5) when the continuous mass distribution is given by $u = f(x)$ at $t = 0$, where $f(x)$ defines the initial land surface elevation. The simplest solution of Eq. (5) is obtained for the initial condition,

$$u = u_0,\ x > 0 \quad u = 0,\ x < 0$$

which means a vertical scarp at $x = 0$. Substituting this into Eq. (3), we have the solution,

$$u(x,t) = \frac{1}{2\sqrt{a\pi t}} \int_0^{+\infty} u_0 \exp\left\{-\frac{(x - bt - \xi)^2}{4at}\right\} d\xi = \frac{u_0}{2}\left\{1 + \operatorname{erf} \frac{x - bt}{2\sqrt{at}}\right\} \tag{6}$$

where the error function erf$x$ is defined by

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x \exp\{-\xi^2\} d\xi$$

The solution shows the simultaneous retreat and subduing of a vertical scarp at $x = 0$ with time as shown in Fig. 2.

A variety of landforms is explained partly in relation to the initial and boundary conditions for the landform equation. Spatial distribution of the erosional coefficients corresponding to lithologic features also concerns such topographic features as structural bench land, cuesta, dyke ridge, etc. The variety

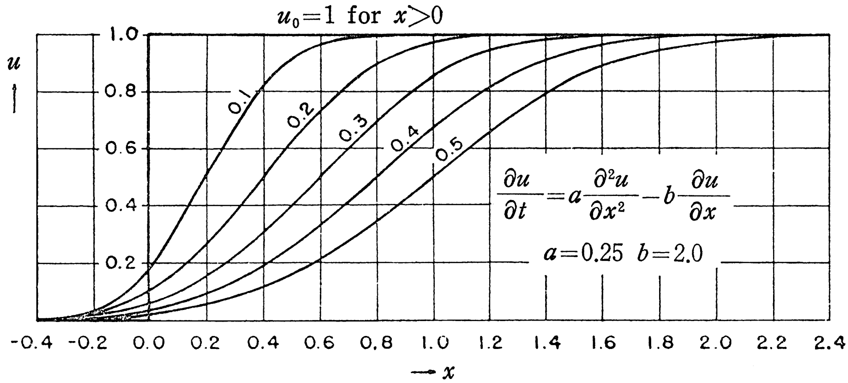

Fig. 2. Retreat and subduing of a vertical scarp at $x = 0$ with time derived from the landform equation. After Hirano (1968).

of morphology is derived too from the nature of tectonic movement which is given by the term $g(x, t)$ added to the right side of Eq. (5). The term $g(x, t)$ defines the rate of material supply above the base level, and possibly it takes the form of $X(x) \cdot T(t)$ corresponding to the separable space-time distribution of such tectonic movement as the case of uplift of block mountains.

LONG-TERM EVALUATION OF EROSIONAL COEFFICIENTS

The coefficients a and b in Eq. (5), the erosional coefficients together as they bring erosion, were called the subduing coefficient and the recessional coefficient, respectively, by Hirano (1968) based on their effects on landform change. A vertical scarp retreats and subdues with time as shown in Fig. 2, as the result of combined effect of these coefficients.

This fact is applied conceptionally to any kind of scarps in order to evaluate a and b quantitatively. Determination of the coefficients over geologic time scale is possible based on the morphometric feature, for instance, of fault scarps for which the fault line as the scaling origin is given, or of terrace scarps for which the age of formation is often given.

Normal distribution given by Eq. (1) concerns the determination of the coefficients a and b. Cumulative curve $\varphi(x)$ of a normal distribution with $\mu = 0$ and $\sigma = 1$ is given by

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left\{-\frac{\xi^2}{2}\right\} d\xi$$

and this has a particular relationship to erf$x$ given by

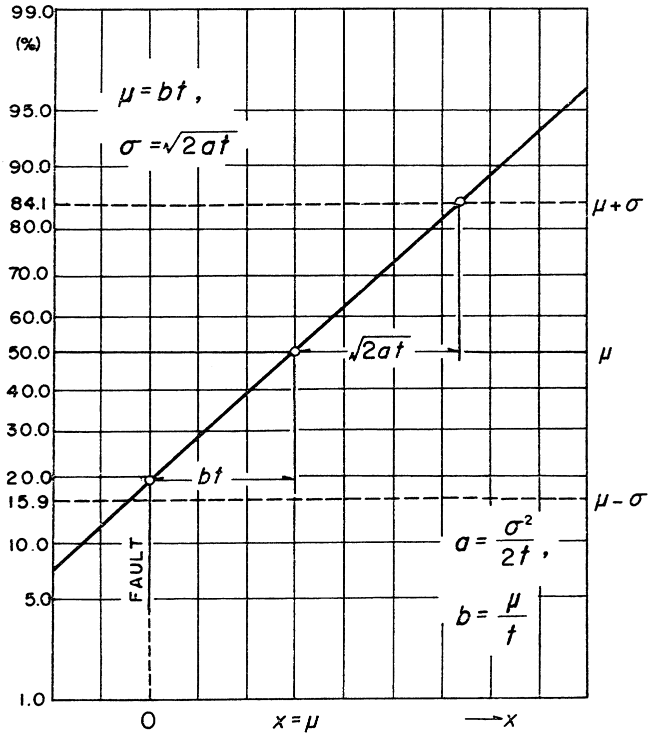

Fig. 3. Morphology of a scarp on a probability paper showing the simultaneous retreat and subduing during a given time span. After Hirano (1972).

$$\varphi(x) = \frac{1}{2}\left\{1 + \mathrm{erf}\left(x / \sqrt{2}\right)\right\}, \text{ or } \mathrm{erf}x = 2\varphi\left(\sqrt{2}x\right) - 1$$

Therefore, we have

$$\sigma = \sqrt{2at} \tag{7a}$$

$$\mu = bt \tag{7b}$$

for Eq. (6) by comparison. Cumulative curve of normal distribution is shown by a straight line on a probability paper (Fig. 3).

Morphometric data on fault scarps even be the volume preservative (Hirano, 1967) or the slope preservative (Hirano, 1972) are often approximated by straight lines on the probability paper with $x = 0$ at the fault line. The quantities μ and σ

Table 1. Estimated erosional coefficients where Rokko and Hira for fault scarps and Ontake for gigantic landslide. Numerals in parentheses were assumed.

Loc.	μ (km)	σ (km)	t	$a(=\sigma^2/2t)$	$b(=\mu/t)$
Rokko(1)	0.22～0.21	0.36～0.27	(10^6years)	0.065～0.036	0.21～0.22
Rokko(2)	0.37～0.38	0.64～0.48	(10^6years)	0.21～0.12	0.37～0.38
Hira	1.09～1.14	0.55～0.64	(10^6years)	0.151～0.205	1.09～1.14
Ontake-1	7	0.5	266～355sec	1.48～1.11x10^{10}	8.3～6.2x10^{11}
Ontake-2	10	1	(308～380sec)	4.15～3.11x10^{10}	(8.3～6.2x10^{11})

*a in m^2/year, b in mm/year.

are thus determined on the paper. If the age t since the scarp formation is known, though it be the Davisian concept on tectonism (Davis, 1912), the constants a and b are scaled, using Eqs. (7a) and (7b) for σ and μ obtained by morphometry.

Determination of t is often difficult for fault scarps as they move continuously or intermittently (the Penckian concept on tectonism as maintained in 1924) but the age of fresh fault scarps in Japanese island is probably 10^6 years or less (almost the later half of the Quaternary) in order of magnitude. As μ and σ for fault scarps have some 10^2~10^3 m in order of magnitude (Hirano, 1967, 1972), the constants b and a are thus to be 0.1~1 mm/year and 0.005~0.5 m^2/year, respectively in geologic time scale as shown in Table 1. Dimensionless parameter σ/μ gives the relative diffusivity to transport distance, and we have σ/μ almost equal to 1. It is a future problem to determine the constants more precisely. Even though, the result concerning the fault scarps suggests the conceptional applicability of Green's function given by Eq. (2) as well as Eq. (5) to the various types of mass transport including rapid mass wasting, as the fault scarps are shaped eventually by the combined or integrated geomorphic processes including the rapid mass movements.

APPLICATION TO PRESENT DAY MASS MOVEMENTS

It is appreciated to evaluate the erosional coefficients for the individual process in the present day space and time, directly from the transportational distance and the degree of diffusion. Diffusion or spreading by an actual event could be almost one-dimensional if the mass moves in a narrow linear channel, but it would be two-dimensional if the mass spreads over an open space. Diffusivity clearly depends on the processes concerned additionally, and several processes are mixed in the actual mass transport. Separation of the processes in addition to the dimensional reduction is, thus, appreciated to evaluate the parameters concerned there if possible.

When the elevation D_* is lost at the origin in one-dimensional profile, we have depositional thickness given by

$$D(x) = \frac{D_*}{2\sqrt{a\pi t}} \exp\left\{-\frac{(x-bt)^2}{4at}\right\} \tag{8}$$

from Eq. (2) for point source at $x = 0$ where the mass of surface material was concentrated before the event. For a planner spreading, debris moves to x-direction but spreads both in x- and y-directions and $(x - bt)^2$ in Eq. (8) is to be replaced possibly with $(x - bt)^2 + y^2$.

If we employ Eq. (8) for the sake of simplicity, we have

$$D(x) = \frac{D_*}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\} \tag{8'}$$

by replacing $\sqrt{2at}$ with σ, and bt with μ, respectively. The intensity of point source D_* can be estimated from the maximum thickness of the accumulated debris along the central profile or from the volume at the source correlative to unit width. The parameters μ and σ are also estimated from the debris thickness distribution after the mass movement.

Here we investigate the case of Mt. Ontake as a representative and effective one where a part of the volcanic slope was destroyed by a gigantic landslide triggered by the 1984 earthquake. The volume in the source area obtained by photogrammetry exceeds 3.4×10^7 m^3 (Nagaoka, 1984). The failed mass moved downward along a channel, and the change of river-bed elevation by the accumulation of debris was detected by precise photogrammetry as shown in Fig. 4A by Okuda *et al*. (1985) and by morphometry from topographic maps in Fig. 4B. Though the debris flowed partly over ridges into other channels, we employ here one-dimensional model for the sake of simplicity.

Two remarkable accumulations of debris are detectable in Fig. 4. Depositional amount along the channel is approximated respectively by a normal distribution as shown in Fig. 4 by the broken lines, though the down stream extension of debris suggests the existence of other minor accumulation units. We have the approximate values of $\sigma_1 = 0.5$ km and $\mu_1 = 7$ km for upstream with $D_{*1} = 65$ m, and $\sigma_2 = 1$ km and $\mu_2 = 10$ km with $D_{*2} = 100$ m for downstream, respectively, where μ was measured from the approximate center of the source area. It is noted that the sum $D_{*1} + D_{*2}$ is almost correlative to the maximum depth 150 m of the landslide.

The time required for debris transport from the source to the confluence of Nigorizawa on the Ohtaki-gawa river shown by 0-km point in Fig. 4 is estimated to be 367 sec to 390 sec, and the average velocity ranges from 26.3 to 19.7 m/sec (Okuda *et al*., 1985), or 8.3 to 6.22×10^8 m/year, depending on the check-point specification. The values give the coefficient b_1 directly, and this is applicable to the first (up-stream) debris mound. From this fact, we have $t_1 = 266.2$ sec to 355.3 sec for $\mu_1 = 7$ km for upper mound, and thus $a_1 = \sigma_1^2/2t_1 = 469.9\text{~}351.8$ m^2/sec,

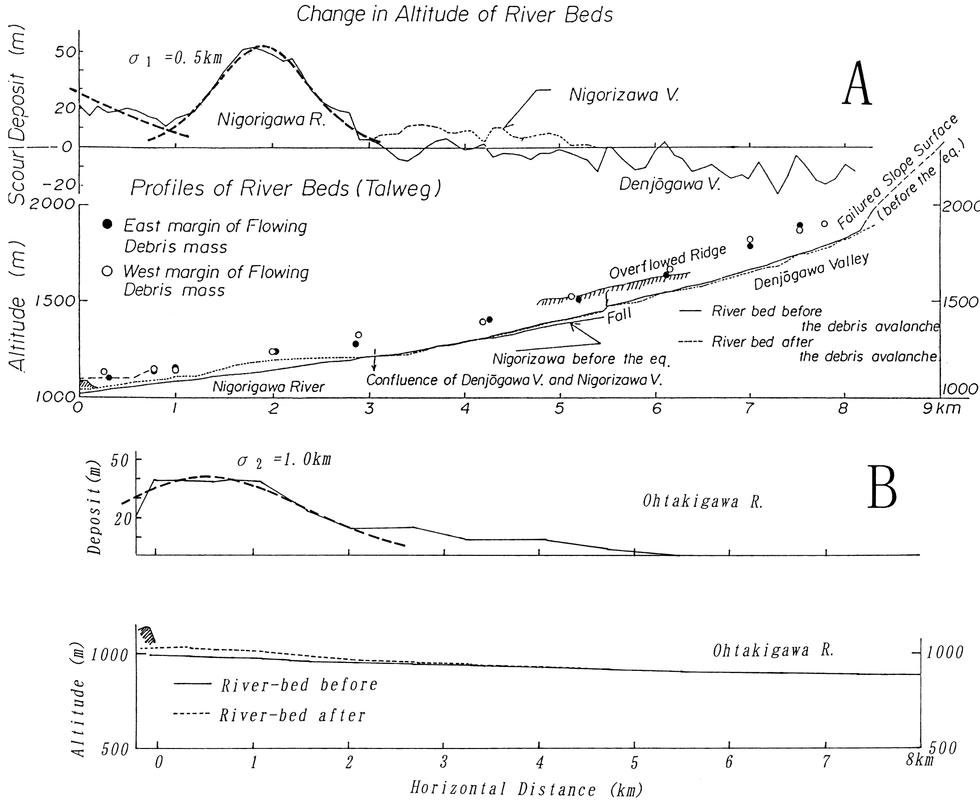

Fig. 4. Gigantic landslide at Mt. Ontake and debris transport where A (above) is simplified from Okuda *et al*. (1985) and B (below) by morphometry from maps. Change of river-bed elevation due to debris accumulation is approximated by a couple of normal distributions.

or 1.48~1.11 × 10^{10} m^2/year.

It is difficult to know exactly when the second (down-stream) debris mound stopped. A possible case is simultaneous freezing under the different moving velocity, and another case is successive freezing one after another presumably moving with almost same velocity. Estimation of the velocity just at the confluence point using super-elevation gives 17 m/sec (Okuda *et al.*, 1985) which can be applied to the down-stream debris mound, and the value suggests the latter case as an approximation. Thus, we have t_2 = 380.2 to 507.6 sec for $b_2 = b_1$, and this brings a_2 = 1315~985 m^2/sec or 4.15~3.11 × 10^{10} m^2/year. The results are summarized in Table 1. It is true that a or b is extraordinarily large. However, the dimensionless parameter σ/μ is 0.07 for the up-stream and 0.05 for the down-stream, and no significant difference is detected.

FURTHER REMARKS

The concept of Green's function is applied to any landform change eventually. However, the long-term erosional coefficients obtained from the fault scarp morphology is much less than those obtained from the present day mass movement. Nevertheless, comparison of the dimensionless parameters, σ/μ, means that the material composing the fault scarps diffuses more easily through long time span. A possible way to overcome the contradictions is a statistical investigation.

It is probable to consider from statistical point of view that the erosional coefficients for wide area over long period give the average values for some ensemble of individual and discrete processes. Statistical investigation concerns the magnitude-frequency relationship and the return period of the events even in the case of landslide, in addition to the differential mobility of the debris characteristic to the respective process.

In order to compare the long-term diffusivity with that for single event, it is noted that a landslide as a source appears at a definite time, to say, $t = \tau$. This means in the statistical investigation that a point source begins to work at $t = \tau$, and that the frequency of source appearence or the return period of landslide has to be taken into account.

Moreover, a definite time span concerns the mass transport. The situation is shown in Fig. 5, where each of respective events has happened discretely over a long period, and has a specific duration time t_i for the movement. The average rate over the long period in which several events are included gives much less value than that of the individual event as shown in Fig. 5 with some exaggeration.

From spatial point of view, the relationship between a single event and a regional average is concerned. The area A_* and the volume V_* of a landslide are known for individual mass transport. Thus we may divide at first the whole area A into the unit area of A_*, and introduce a grid system. In other words, the whole area A is consists of n unit areas, and we have

$$A = nA_* \tag{9}$$

Probability of occurrence of an event once over the whole area brings the

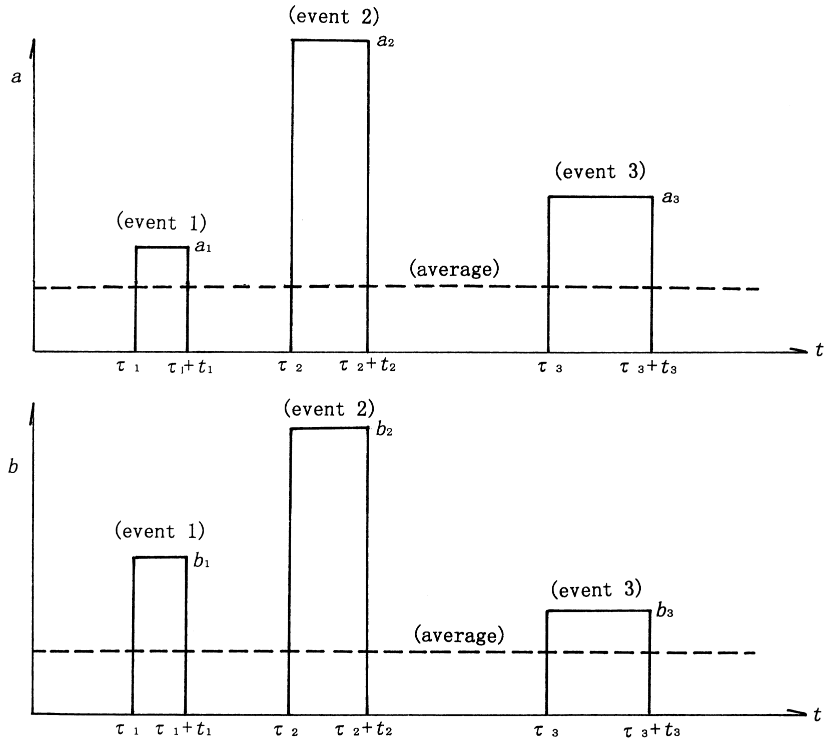

Fig. 5. Schematic relationship between the discrete unit events and the long-term average in the evaluation of coefficients. Coefficients obtained by application of Green's function for individual unit event can be extraordinarily larger than the average value.

frequency of $1/n$ for a grid point spatially. If the recurrence period of the event is m years, we have the frequency of $1/m$ for a year chronically at a grid. Therefore, we have the probability $1/mn$ for a year at a grid point. Lowering of the elevation by an event is thus given by

$$\delta u = D/mn \tag{10}$$

a year at a grid point in average, where D is the depth given by V_*/A_*.

If we apply this to the case of Ontake, the total area of Japanese Islands is ca. 3.7×10^5 km^2 though the area of mountainous region is less than A given here, and the area A_* is 0.65 km^2 for the Ontake landslide approximately. Thus, we have $n = 5.7 \times 10^5$. The landslide of Ontake class is estimated to be twice (another identical case is Mt. Bandai in 1888) or 4 times (with the additional Totsukawa hazard in 1889 and Hiedayama landslide in 1911) for recent 100 years in Japan, and this means $m = 50 \sim 25$. We have thus $1/mn = 0.36 \sim 0.70 \times 10^{-7}$. The average depth of Ontake landslide is $D = 3.4 \times 10^7$ m^3/0.65 km^2 = 52.3 m, and we have the

Table 2. Ontake landslide and parameter estimation where b is for $|du/dx| = 0.3$ (total area of Japanese Islands, A; 3.7×10^5 km^2).

Volume (V_*);	3.4×10^7 (m^3)	
Area (A_*);	0.65 (km^2)	
Average depth (D_*);	52.3 (m)	
Frequency (f_*);	2/100	or 4/100 (years)
$\lvert \delta u/\delta t \rvert$	1.9×10^{-6}	or 3.7×10^{-6} (m/year)
recessional coefficient (b)	0.0063	or 0.012 (mm/year)

average lowering of $\delta u = 0.63 \sim 1.2 \times 10^{-2}$ m a year for the whole area.

In order to translate δu to b based on Eq. (5), the slope gradient $\partial u/\partial x$ must be introduced, even if $\partial^2 u/\partial x^2$ is very small and negligible. Assuming $I = |\partial u/\partial x| = 0.3$ (=16.7 deg.) in average, we have $b = 6.1 \times 10^{-3}$ mm/year from $b = |\partial u/\partial t|/I$, if the term including $\partial^2 u/\partial x^2$ is neglected (Table 2). Choice of another possible value of $\partial u/\partial x$ does not bring the remarkable change of order of magnitude.

Another approach based on the travel distance is possible, which may be more significant statistically than the instantaneous moving velocity, by analogy to Brownian movement evaluated by the resultant mean free path of molecules. Having distribute the value of $\mu = 7 \sim 10$ km at Mt. Ontake to all grid points and taking the average over the recurrence period, we have $b = 0.25 \sim 0.7$ mm/year at each grid for $1/mn = 0.36 \sim 0.70 \times 10^{-7}$. The value is correlative to that for fault scarps.

We face here some discrepancy especially in the first estimation that the long-term value for fault scarps is more than 10 times larger than the value in the first estimation. The discrepancy may comes first from the estimation of parameter values including A. Morphometric data based on digital elevation model over Japanese Islands may supply more improved estimates of slope gradient and curvature (Laplacian). The second reason of discrepancy comes from the magnitude-frequency relation of mass movement as discussed by Hirano and Ohmori (1989). Especially the latter concerns the total volume estimation by mass movements over long period, and it exhibits the fractal nature that the frequency decreases with the increasing magnitude (area or volume). Therefore, the geomorphic processes with small magnitude but high frequency may bring the superposition of δu, and b larger than those obtained here may be expected.

CONCLUSION

Any kind of mass movement on the slope is characterized by the transport distance and the degree of scattering of the debris. The debris thickness is approximated by a normal distribution eventually. Based on this fundamental features, it is possible to introduce a Green's function identical to the normal distribution with $\mu = bt$ measured form the source and with $\sigma = \sqrt{2at}$. The Green's function in broader sense explains the simultaneous transportation and diffusion of a unit mass at a point source.

We of course face to the problem that the debris has to distribute from –∞ to +∞ logically if the normal distribution is applied. This is different from the actual debris distribution after the mass movements. It is well known however that the normal distribution is an approximation to the binomial distribution which shows a definite range of distribution. The normal distribution or the identical Green's function converges quickly leaving from the mean though it covers continuously from –∞ to +∞, and the convergence as well as the continuity plays such an important role as a special function to be a singular solution of partial differential equation from the analytical point of view.

The partial differential equation, namely the landform equation with the erosional coefficients, corresponding to the Green's function discussed here explains the features of landform evolution systematically. It is possible thus methodlogically to apply the concept to long-term landform change and to present day processes. In other words, the Green's function of mass transport gives a way to integrate the present day individual processes over the space and time on the basis of the corresponding landform equation.

Conclusively, Green's function for a point source can be a common scale to measure various kinds of present day mass transports and to evaluate their integrated effects over the long time and the large space. Application of this method to many more cases and evaluation of the features of mass transport processes in terms of the erosional coefficients are appreciated in the future, in addition to the precise statistical and kinetic investigations covering the individual mass movement. Of course, the limitation of one-dimensional treatment should be emphasized too, as the actual process is not the simple linear diffusion but it occurs on rugged topographic surface.

REFERENCES

Carslaw, H. S. and Jaeger, J. C. (1959) *Conduction of Heat in Solid* (2nd ed.). Oxford Univ. Press, London, 510 pp.

Culling, W. E. H. (1960) Analytical theory of erosion: *Jour. Geol.*, **68**, 336–344.

Davis, W. M. (1912) *Die erklarende Beschreibung der Landforemen*. Teubler, Leipzig, Berlin, 565 pp.

Hirano, M. (1966) A study of mathematical model of slope development: *Geogr. Rev. Japan*, **39**, 36–48 (in Japanese with English abstract).

Hirano, M. (1967) Discussion on the two-dimensional morphometry of fault scarps: *Chikyu-kagaku*, **21-3**, 11–21 (in Japanese with English abstract).

Hirano, M. (1968) A mathematical model of slope development: *Jour. Geosci., Osaka City Univ.*, **11**, 13–52.

Hirano, M. (1972) Quantitative morphometry of fault scarp with reference to the Hira Mountains, Central Japan: *Japan Jour. Geol. Geogr.*, **XLII** (Nos. 1–4), 85–100.

Hirano, M. (1975) Simulation of developmental process of interfluvial slopes with reference to graded form: *Jour. Geol.*, **83**, 113–123.

Hirano, M. (1976) Mathematical model and the concept of equilibrium in connection with slope shear ratio: *Z. Geomorph.*, Supple. Bd., **25**, 50–71.

Hirano, M. (1990) Mass diffusion models in geomorphology and the related problem: *Trans. Japan Geomorph. Union*, **11-3**, 191–215 (in Japanese with English abstract).

Hirano, M. and Ohmori, H. (1989) Magnitude-frequency distribution for rapid mass movements and its geomorphic implications: *Trans. Japan Geomorph. Union*, **10**, 95–111 (in Japanese with English abstract).

Nagaoka, M. (1984) Disasters by Naganoken-seibu earthquake: *Sokuryo*, **34** (No. 12), 22–28 (in Japanese with English abstract).

Okuda, S., Okunishi, K., Suwa, H., Yokoyama, K. and Yoshioka, R. (1985) Restoration of motion of debris avalanche at Mt. Ontake in 1984 and some discussions on its moving state: *Bull. Disas. Prev. Inst., Kyoto Univ*., **28**(B-1), 491–504 (in Japanese with English abstract).

Penck, W. (1924) *Die morphologische Analyse*. J. Engelhoruns Nachf., Stuttgart, 283 pp.