Chapter 2

Generation and Nonlinear Evolution of Cometary Waves

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Abstract. The observations carried out in 1985/1986 by several space missions (ICE, VEGAs 1 and 2, Suisei, Sakigake, Giotto) in the environments of comets Giacobini-Zinner and Halley spurred intense cometary plasma wave research. The interpretation of these in situ particle and field measurements fostered investigations on (among other topics) wave generation that, leaving aside the inherently nonlinear (but not unrelated) problem of the eventual formation of a cometary bow shock wave, explored the free energy available in two specific features of the velocity distributions of the newborn particle populations: their parallel (with respect to the IMF direction) drift in the solar wind frame and perpendicular ring-like organization. Analytical and simulation works looked into the influence of the solar wind and cometary newborn parameters on the characteristics of the instabilities and the ensuing, or associated (as evidenced by wave observations), nonlinear phenomenology. Comprehensive reviews have described the results obtained in this cometary wave research until 1992 and identified outstanding problems warranting further attention. Here, only a cursory revisit to the Giacobini-Zinner/Halley era of wave generation and nonlinear evolution shall be made: rather, attention shall be predominantly focussed on the new implications to cometary wave research of the recent Giotto encounter with comet Grigg-Skjellerup on July 10 of 1992. The three visited

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comets, starting with their gas production rates, had different characteristics that showed in the in situ observations. However, with the important exception of the GS encounter, the interpretation of the wave activity could be made in terms of common basic generation mechanisms adapted to the local properties of the plasma medium. Yet, new aspects emerged in the last Giotto cometary mission: the smaller gas production rates yield a scale length for the neutral gas density that is not (much) larger than the gyration distance of a heavy newborn ion (estimated by the product of the solar wind speed and the ion cyclotron period). As a consequence of this inhomogeneity, the velocity distribution of the heavy newborn ions exhibits gyrophase organization, i.e. nongyrotropy. This new source of free energy was not investigated in the context of the Giacobini-Zinner and Halley encounters. Since (i) the GS Giotto observations strongly suggest that nongyrotropy plays a prominent role in wave generation as the comet nucleus is approached and (ii) its stability characteristics have only seldomly been analyzed, the exposition shall be mainly centered on the wave generation effects of particle populations with gyrophase organization. Nongyrotropy can linearly couple wave eigenmodes, enhance previously existing instabilities and destabilize otherwise thoroughly passive media. Both analytical and simulation results shall be presented to illustrate these effects.

2.1 Introduction

Cometary research supported by in situ measurements started on December 27, 1984 with the attempt by the AMPTE mission to create an artificial comet through the release of a cloud of barium gas in the solar wind [Gurnett et al., 1985]. In quick succession followed the observations carried out in 1985 and 1986 by ICE, VEGA 1, VEGA 2, Suisei, Sakigake and Giotto in the environments of comets Giacobini-Zinner (G-Z), visited only by ICE, and Halley, target of all these spacecrafts. After a hiatus of several years, and a few reactivation/hibernation and orbit-control manoeuvres, Giotto was successfully aimed at comet Grigg-Skjellerup (G-S), reaching closest approach (∼200 km) on July 10, 1992 [Grenseemann and Schlemm, 1993].

For the purpose of this article, it is convenient to divide the cometary studies into before and after G-S. The initial G-Z and Halley measurements were reported in Science (April 1986), Geophysical Research Letters (March–April 1986), Nature (May 15–21, 1986), ESA SP-250 (December 1986), and Astronomy and Astrophysics (187, 1987). They spurred intense experimental and theoretical investigations of cometary wave activity that have been described in several reviews [Lee, 1989; Scarf, 1989; Gary, 1991, Brinca, 1991; Roberts and Goldstein, 1991; Tsurutani, 1991a; 1991b; Tsurutani, 1992] where outstanding problems warranting further attention have
also been identified. In contrast, interpretation of the G-S observations has just started, with the first research results published during 1993 and a special section of the December 1993 issue of the *Journal of Geophysical Research (Space Physics)* dedicated to the Giotto encounter with comet G-S. Understandably, only a cursory revisit to the G-Z and Halley era of wave generation and non-linear evolution shall be made here: emphasis shall be predominantly focussed on the novel implications to cometary wave research of the Giotto measurements in the G-S environment.

The density of the neutral particles originated in the nuclei at cometarycentric distances beyond the bow wave are small with respect to the solar wind density. However, the ionization of these neutrals corresponds to the abrupt injection of free energy into the solar wind (frame) and can feed a large variety of instabilities. The generated waves play an important role in the pickup of cometary ions, ensuing (mass loading) deceleration of the solar wind, and the eventual formation of shock fronts; the associated wave-particle interactions in this collisionless magnetoplasma contribute to the heating and acceleration of plasma particles.

The free energy sources lie in the cometary particles sublimated from the ices in the nuclei. Their speeds in the inertial (and comet) frame are very small when compared to $V_{\text{SW}}$, and have correspondingly small thermal spreads. When they become ionized, the newborn charged particles are injected into the solar wind frame with a velocity close to $-V_{\text{SW}}$. They describe helical trajectories about the IMF, with perpendicular (with respect to $B_0$) speeds of $\sim |V_{\text{SW}} \sin \alpha|$ and parallel drifts of $\sim |V_{\text{SW}} \cos \alpha|$, where $\alpha$ defines the relative orientation of the solar wind velocity $V_{\text{SW}}$ and the ambient IMF, $B_0$. A simplified, commonly adopted model of their distribution in velocity space is thus a drifting ring, with limiting cases of pure rings ($\cos \alpha = 0$) or beams ($\sin \alpha = 0$). This approach and its variations were exhaustively applied in analytic and simulation studies of cometary wave generation before G-S, exploring the free energies residing in the parallel drifts and the "temperature" anisotropies of the newborns, as modulated by the all important $\alpha$ angle.

The last Giotto cometary encounter evidenced qualitatively new aspects in the free energy picture assumed before G-S: the smaller gas production rates of this comet yield a scale length for the neutral gas density that is not (much) larger than the gyration distance of a heavy newborn ion (estimated by the product of the solar wind speed and the ion cyclotron period). As a consequence of the inhomogeneity, the velocity distribution of those particles exhibits gyrophase organization, i.e. the heavy newborn ions are nongyrotropic [Neubauer *et al.*, 1993]. The emphasis of the exposition shall lie on the description of potential effects on wave generation of this new source of free energy.

Several examples of generation and nonlinear evolution of cometary
waves studied in the context of the G-Z and Halley encounters, together with the discussion of some associated unresolved problems, precede the presentation of the potentialities of nongyrotropy in the realm of wave activity interpretation. Both analytical and simulation results shall illustrate that gyrophase organization can linearly couple wave eigenmodes, enhance previously existing instabilities and destabilize otherwise thoroughly passive media.

2.2 Wave Stimulation

The parameter space available to study the stability of the cometary environment includes the characteristics of the solar wind, the newborn particles and the IMF. Not surprisingly, the wide choice of potential media translates into a large number of instabilities fed by the free energy available in the cometary particles. Many of the anticipated instabilities have not been positively identified in the cometary space missions: not only the necessary discriminatory observations might be impossible with the available experimental setups, but many of the simplifying hypotheses adopted in the theoretical analyses might not find adequate correspondence with the real cometary environment. More importantly, the converse is also true: many features of the observed wave activity have not yet been convincingly explained, meaning that there exists ample room for theoretical analyses aimed at the interpretation of the experimental cometary wave data.

For the purpose of this review, wave generation and nonlinear behavior shall be associated with linear instabilities and their evolution. The domain thus defined, even restricted to cometary conditions, is vast and, as already stressed, has been the object of several reviews connected to the G-Z and Halley era. An exhaustive approach is thus unwarranted here; rather, for the period before G-S, a few examples of wave activity observations with implications on the linear and nonlinear behavior of the medium shall be discussed.

2.2.1 Prologue

For well defined solar wind and cometary populations, the IMF orientation (and its magnitude to a smaller extent) with respect to \(V_{SW}\), denoted by \(\alpha\), determines the stability properties of the cometary environment. Leaving aside for later consideration the eventual (free energy associated with the) nongyrotropy of the newborn particles, the free energy residing in the drift ("beam-like") and cyclotron ("ring-like") motion of the cometary populations (usually analyzed in the solar wind frame) is able to stimulate several types of instabilities: electromagnetic or electrostatic, parallel or oblique, resonant (kinetic) or nonresonant (fluid), high (with respect to
the lower hybrid), intermediate or low frequency. Ignoring the types of ion species and the distributed spatial production of cometary newborns, there exist similarities between the free energy sources of the cometary environment (newborn beams) and of the foreshock region (reflected particle beams); with a common background medium (solar wind with $\beta \sim 1$), it is not surprising to find common wave phenomenology in the two media [Tsurutani, 1992]. This aspect may prove useful for the cometary research because of the numerous observations carried out in the foreshock region.

Oversimplifying a complicated situation, the theoretical analyses of the simple beam-ring model suggest that small $\alpha$ (dominant "beam") predominantly excites resonant right-hand instabilities (ion beams faster than the costreaming magnetosonic wave); other anticipated instabilities, such as the fluid firehose instability, do not seem to have been identified in the cometary observations. Large $\alpha$ (dominant "ring") tends to favor kinetic left-hand and fluid mirror instabilities fed by anisotropy; interestingly, the cometary wave observations before G-S have encountered mirror modes but not the left-hand waves expected for large $\alpha$.

This later situation constitutes one of the existing puzzles but it is not a monopoly of the cometary environment. Several attempts have been made, for example, to interpret somewhat similar occurrences in the magnetosheath, albeit here the left-hand waves are also observed (the linear growth rates and numerical simulations of the two competing modes would suggest an inexistent dominance of the left-hand waves). Price et al. (1986) have suggested that minor ion species could affect the growth of the left-hand ion cyclotron waves; Gary et al. (1993) found that the mirror mode dominates low anisotropy, high ion $\beta$ plasmas in which the helium density is a few percent of the total ion density. Tsurutani et al. (1989a) propose that measurable left-hand wave amplitudes require long convective growth times, i.e. need large intervals with reasonably steady free energy sources available that may not occur in environments with turbulent ambient fields. This suggestion is (partially) supported by the observation of left-hand wave activity at G-S [Neubauer et al., 1993], where the Giotto encounter occurred during a continued interval of large $\alpha$ (and small Alfvénic Mach numbers, due to the unusually intense IMF). Another possibility, particularly relevant to the cometary situation, has to do with the physical nature of the two instabilities. The ion cyclotron anisotropy instability that generates the left-hand waves is kinetic (resonant) but the mirror instability is fluid (nonresonant); in environments with particle replenishment (e.g. continuous generation of newborns at comets), the recycling of ions with the right (for the instability to occur) anisotropy is detrimental to the growth of the kinetic instability, but benefits the mirror mode growth, as clearly shown by McKean et al. (1992a). Whereas all the newly injected particles contribute to the macroscopic parameters that determine the mirror fluid
instability, only those with adequate phase at resonance are able to transfer part of their energy to the left-hand mode; also, in the ion recycling, some of the removed particles are the few critically resonant previously responsible for the growth of the kinetic instability.

2.2.2 Examples of wave generation

Observation of the field power spectra detected in the cometary environments clearly shows the existence of wave activity well above the levels expected in the ambient solar wind. In particular, the occurrence of peaks around the cyclotron frequency of the dominant (water group) newborn heavy ions (and, sometimes, of its harmonics [Glassmeier et al., 1989]) strongly hints at resonant excitation fed by the cometary ions.

Tsurutani and Smith (1986a) first pointed out that satisfaction of the resonance condition

$$\omega_r - k \cdot v_{pa} \pm n\Omega_s = 0 \; ; \; n = 0, 1, 2, \ldots$$ (1)

(where $\omega_r$ and $k$ denote the wave real frequency and wavevector, and $v_{pa}$ and $\Omega_s$ stand for the resonant particle parallel velocity and cyclotron frequency) implies that the measured spacecraft frequency spectrum is approximately centered at (multiples of) the cyclotron frequency of the resonant newborn species. Caveats to this assertion are, however, in order [Brinca, 1991]; for example, resonant water group ions can excite modes with spacecraft frequencies of the order of the proton cyclotron frequency.

Figure 1, taken from Tsurutani and Smith (1986b), shows the power spectra of the magnetic field components measured by ICE at G-Z and, for comparison, includes a "typical" spectrum of an "active" solar wind. The peak in the three power spectra is centered around $\Omega_{H_2O}/2\pi \simeq 10^{-2}$ Hz, suggesting resonant cometary wave excitation by the water group newborn ions. The ion beams with parallel velocities larger than the costreaming magnetosonic wave phase velocities stimulate the right-hand resonant beam instability. The waves thus generated are convected by the solar wind past the spacecraft where they impart a Doppler-shifted left-hand polarization signature.

Another puzzle in the cometary wave activity is the inexistence of spectral peaks in the neighborhood of the proton cyclotron frequency since the disassociation reactions lead to more numerous hydrogen atoms than parent water molecules; Gary et al. (1988) show that pitch angle scattering is faster for smaller lighter ions, thus achieving faster saturation at weaker wave amplitudes, but Tsurutani (1992) points out that, in spite of the faster scattering, proton cyclotron waves generated by ion beams of similar relative densities and velocities are easily observed in the foreshock region. Isolated instances of wave packets near the local proton cyclotron
frequency have been observed, nevertheless. Tsurutani et al. (1989b) report ICE measurements at G-Z of single-cycle magnetic pulses displaying clear compressional components in regions where $\alpha \sim \pi/2$, and Mazelle and Neubauer (1993) present Giotto observations at Halley of similar wavepackets but without significant compression, almost parallel propagation and at much smaller $\alpha$. (With respect to the existence, or nonexistence, of wave
activity with spectral signature at the spacecraft frame near the proton cyclotron frequency, there is a controversy on the possible observation of proton cyclotron waves at G-Z resonantly driven by the newborn protons via the right-hand ion beam instability [Tan et al., 1993; Le and Krauss-Varban, 1993; Tan and Mason, 1993].)

Under a set of distinct conditions (different comet, spacecraft, and wave mode), Fig. 2 depicts observations carried out by Giotto at Halley [Glassmeier et al., 1993] showing an anticorrelation between the magnetic field magnitude and electron density that has been associated with a fluid mirror mode. Here the $\alpha$ angle is very large and the pressure anisotropy brought about by the ring distribution of the newborn cometary heavy ions, conjugated with the high $\beta$ in the region, can drive the mirror instability, in agreement with results obtained in numerical simulations [Price, 1989; McKean et al., 1992b] and analytically [Wu et al., 1988]. Similar magnetic

![Graph showing magnetic field amplitude, electron density, and magnetic field elevation angle outside Halley's pileup region (from Fig. 4 of Glassmeier et al., 1993).]
structures were also observed at Halley by the VEGA spacecrafts [Russell et al., 1991].

2.3 Wave Evolution

The occurrence of nonlinear phenomenology is a natural consequence of the evolution of the instabilities. The systematization of the ensuing possibilities is, however, a daunting task: quoting from Carl von Weizsäcker, "The study of the Physics of nonlinear phenomena is like the study of the Zoology of nonelephants". Here, guided by the cometary field observations where one of the striking features is the existence, specially at G-Z, of both high-level turbulence and coherent wave events, we shall again resort to a few illustrative examples of nonlinear phenomenology detected in the G-Z and Halley environments. The frequent occurrence of coherent waveforms at G-Z might have to do with the smaller scale lengths imposed by the smaller gas production rates that hindered a fuller nonlinear evolution (including cascade, wave-wave interactions) towards a Kolmogorov-like spectrum. (The G-S observations, that also include coherent events, have been recently published; besides the implications of lower gas production rates, small Alfvénic Mach number and extended intervals with large $\alpha$, the most original results are associated with nongyrotropy, to be analyzed in some detail below.)

2.3.1 On the nonlinear evolution

A newborn ion beam can drive a right-hand resonant (ion/ion) instability that, as already stressed, plays a prominent role in the cometary phenomenology. The ensuing evolution of the growing waves illustrates the bewildering variety of possible nonlinear paths.

Provided that a few assumptions underlying the theory of weak turbulence are satisfied, the quasi-linear (QL) description yields helpful information on the unfolding of the instability. Use of the QL theory requires small perturbations in the plasma (wave energy densities much smaller than particle energy), a sufficiently wide and dense wave spectrum, and growth rates small with respect to the real frequencies (e.g. Stix, 1992). Then, describing the growing perturbations as a band of modes, its effect on the particle distribution function is to produce velocity-space diffusion in the vicinity of the resonant velocities (with an efficiency proportional to the sum of the squares of the mode amplitudes); the diffusion gradually modifies the distribution in the resonance region, driving the instability growth rate to zero. The QL approach has been applied to the cometary problem and can be used to estimate the level of turbulence generated by the newborns (e.g. Lee and Gary, 1991). Simpler analyses invoking the same right-hand resonant
instability, but looking at the energies associated with the initial and final cometary particle distributions to estimate the stimulated wave energy, have been used to calculate the \( k \) spectrum of the cometary-ion-generated turbulence, providing an almost complete quantitative explanation of the spectrum observed by Giotto at Halley [Huddleston and Johnstone, 1992]. (In this respect, I would like to point out that this approach might lead to erroneous implications: one can start from a thoroughly passive maxwellian distribution without free energy and, introducing gyrophase organization that keeps the kinetic energy of the distribution constant, destabilize the medium, i.e. the free energy available in the new nongyrotrropic particle distribution can drive wave growth.)

The original cometary "ring-beam" can, however, evolve in other ways. The excited electromagnetic waves can steepen to form shocklets that might form cometary shocks [Omidi and Winske, 1990], when they propagate obliquely to \( \mathbf{B}_0 \), and bring about pulsations, when they are approximately field aligned [Akimoto et al., 1993]. The former wave evolution is perhaps of more interest to the cometary environment and is responsible for some observations of coherent waveforms, albeit posing another puzzle at the very outset of the evolution.

The (magnetosonic) wave steepening can occur because two factors coexist: the (nonlinear) refractive index of the medium depends on wave amplitude (stronger waves travel faster) and the propagating wave has a varying amplitude. Thus, circularly polarized, parallel propagating electromagnetic waves cannot steepen (their wave amplitude is constant); at oblique propagation, the elliptical or linear polarizations already yield the necessary varying amplitudes. However, and this is the puzzling situation mentioned above, linear analysis of the right-hand resonant instability driven by the (e.g. cometary) ion beam shows maximum growth at parallel propagation: how did the waves acquire the oblique propagation? Again, similar problems arise in the Earth's foreshock; Hada et al. (1987) suggest that wave refraction originated in the medium inhomogeneity brings about the required obliquity.

2.3.2 Examples of coherent nonlinear behavior

Tsurutani et al. (1987) provide examples of coherent waveforms observed near the bow shock of comet G-Z where it is clear the steepening of the magnetosonic mode resonantly driven by the newborn water group ions; higher frequency whistler waves are also evident at the steepened, leading edge of the magnetosonic waves, as reproduced here in Fig. 3. Omidi and Winske (1990) use numerical simulations carried out with an electromagnetic hybrid code (fluid electrons, particle ions) to investigate the nonlinear evolution of oblique low-frequency magnetosonic waves resonantly stimu-
Figure 3: Giacobini-Zinner examples of steepened waves and high-frequency wave packets (or partial rotations) at the steepened edges (from Fig. 4 of Tsurutani et al., 1987).

lated by ion beams. The simulation results are able to reproduce many of the observed features, including the steepening and the whistler trains. The steepening process is associated with the coherent generation of a broad spectrum of waves on the magnetosonic whistler branch. Because the whistler wave velocities increase with frequency (when the frequency is much smaller than the electron cyclotron frequency), the high end of the spectrum thus generated propagates ahead of the steepened front and forms a high-frequency wave packet.

A distinct type of high-amplitude coherent waveform with puzzling characteristics was detected by ICE near the outbound bow shock/wave of G-Z [Tsurutani et al., 1990]. The nonlinear solitary magnetic pulses have large compression ratios, are nearly circularly polarized (right-hand in the spacecraft frame, independently of their direction of propagation relative to the solar wind) and highly oblique (typical angles of $55^\circ$ – $75^\circ$ between
k and B_0). Existing theories of finite amplitude Alfvén waves do not fit the observed properties, albeit solutions [Hada et al., 1989] of the DNLS (derivative nonlinear Schrödinger) evolution equation adhere to some of the experimental characteristics (oblique solitons with right-hand circular polarization). Other evolution equations for finite amplitude Alfvén waves can be derived for distinct parameter regimes (e.g. Hada, 1993) and further research in this area might help the interpretation of the observed magnetic pulses.

2.4 Occurrence of Nongyrotropy

Nongyrotropic particle populations are frequently encountered in space plasmas. Their velocity distributions in the perpendicular (with respect to the ambient magnetic field, B_0 = B_0 \hat{x}) plane depend on the gyrophase angle \( \phi = \tan^{-1}(v_z/v_y) \). The ISEE satellites provided the first observations of gyrophase organization (in the ion population): several Earth radii upstream of the bow shock [Gurgiolo et al., 1981] and in the foreshock region [Eastman et al., 1981]. Ion nongyrotropy has also been observed downstream in the magnetosheath [Schopke et al., 1990], in the space shuttle environment [Cairns, 1990] and, more recently, in the G-S Giotto cometary encounter [Coates et al., 1993]. [Anderson et al., 1985] report ISEE 1 and 2 measurements indicating the existence of nongyrotropic electrons both upstream and downstream of the bow shock.

Mechanisms bringing about gyrophase organization include the filtering action associated with particle reflection and transmission at the bow shock and the creation of newborns in (strongly) inhomogeneous magnetoplasmas [Neubauer et al., 1993]; Burgess (1987) used simulations to show that backstreaming ions are organized in gyrophase by the reflection process at oblique shocks. Also, the nonlinear evolution of the fundamental cyclotron resonance brings about nongyrotropy through phase-bunching, a magnetic trapping mechanism invoked, for example, in triggered whistler emissions (e.g. Dysthe, 1971), electromagnetic wave energization of heavy ions [Mauk, 1982], and gyrophase organization of upstream beam ions [Hoshino and Terasawa, 1985].

Nongyrotropy is also relevant to other (terrestrial) applications. For instance, Sudan (1965) and Fruchtman and Friedland (1983) studied the wave dispersion characteristics associated with nongyrotropic electron populations in fusion plasmas and free electron lasers, respectively.

2.5 Consequences of Nongyrotropy

The G-S comet differed from G-Z and Halley during their encounters with the spacecrafts in the gas production rates (much lower gas production
rates brought about smaller characteristic scale lengths), in the Alfvenic Mach numbers (since the G-S encounter occurred with intense IMF, the associated $V_{SW}/v_A$ ratios were lower) and in the $\alpha$ angle (Giotto consistently detected large values). These features have implications on the cometary wave activity [Glassmeier and Neubauer, 1993; Neubauer et al., 1993; Motschmann and Glassmeier, 1993]: the last two tend to favor (hinder) the stimulation of left-hand (right-hand) waves, as evidenced by the observations, and the first one enhances the inhomogeneity of the medium that favors the production of nongyrotropic distributions for the newborn particles.

The Giotto observations suggest that nongyrotropy markedly influenced the wave phenomenology near the G-S comet. Our concern here is to provide the basis for further investigation of this topic, recognizing that the present development of the nongyrotropic stability theory is insufficient to fully comprehend the novel G-S findings.

2.5.1 Unperturbed distributions

Because attention shall be restricted to parallel wave propagation, the description of configuration space requires one ($x$) coordinate for the spatial dependences. Adopting cylindrical coordinates in velocity space ($v_x, v_\perp, \phi$), the unperturbed (by the waves) nongyrotropic distribution function satisfies the Vlasov equation

$$\left( \frac{\partial}{\partial t} + v_x \frac{\partial}{\partial x} - \Omega_b \frac{\partial}{\partial \phi} \right) F_{ob}(v_x, v_\perp, \phi, x, t) = 0 \quad (2)$$

where $\Omega_b = q_b B_0/m_b$ denotes the signed cyclotron frequency of the nongyrotropic species.

The dependency of the nongyrotropic population on the gyrophase $\phi$ precludes the unperturbed $F_{ob}$ to be both constant in time and homogeneous in space. Solutions of (2) allow for dependence on $t$, $F_{ob} = F_{ob}(v_x, v_\perp, \phi + \Omega_b t)$, or $x$, $F_{ob} = F_{ob}(v_x, v_\perp, \phi + \Omega_b x/v_x)$, or both, $F_{ob} = F_{ob}(v_x, v_\perp, \phi + \Omega_b t, \phi + \Omega_b x/v_x)$. Following Sudan (1965), we assume an unperturbed $F_{ob}$ that is spatially homogeneous. Because any dependence on the gyrophase has to be $(2\pi)$ periodic, we take

$$F_{ob} = F_{ob}(v_x, v_\perp, \phi + \Omega_b t) = \sum_{n=-\infty}^{\infty} G_n(v_x, v_\perp) \exp(-i n (\phi + \Omega_b t)) \quad (3a)$$

where the Fourier coefficients $G_n$ are obtained from

$$G_n = \frac{1}{2\pi} \int_{2\pi} F_{ob} \exp(i n (\phi + \Omega_b t)) d(\phi + \Omega_b t) \quad (3b)$$
2.5.2 Wave and dispersion equations

Linear solution of the Vlasov and Maxwell equations [Brinca et al., 1993a] yields the matrix wave equation

\[
\begin{pmatrix}
m_{++} & m_{+x} & m_{+-} \\
m_{x+} & m_{xx} & m_{x-} \\
m_{-+} & m_{-x} & m_{--}
\end{pmatrix}
\begin{pmatrix}
\bar{E}_+(\omega, k) \\
\bar{E}_x(\omega - \Omega_b, k) \\
\bar{E}_-(\omega - 2\Omega_b, k)
\end{pmatrix}
= 0
\] (4)

where the matrix components \(m_{rs}\) are defined in the Appendix of Brinca et al. (1993a), \(\bar{E}_s\) denotes the (Fourier transformed) complex amplitudes of the wave electric field, and the + and − subscripts identify the (“gyrating”) combinations

\[
\begin{pmatrix}
\gamma \\
\epsilon_y \\
\epsilon_z
\end{pmatrix}
= \begin{pmatrix}
\gamma \\
\epsilon_y \pm i\epsilon_z
\end{pmatrix}/2
\]

Nontrivial solutions imply the dispersion equation \(\text{Det} |m_{rs}| = 0\).

Two effects of the introduction of nongyrotropy became analytically clear (the underlying physics shall be discussed below). The first relates to the frequency shifts occurring in the arguments of the field components: the frequencies of the complex amplitudes of the eigenmode components of the gyrotrropic medium differ by \(\Omega_b\) and \(2\Omega_b\). The second effect is associated with the existence of nondiagonal terms in the matrix \(m_{rs}\) and brings about linear coupling among the components \(\bar{E}_+(\omega, k), \bar{E}_x(\omega - \Omega_b, k)\), and \(\bar{E}_-(\omega - 2\Omega_b, k)\): nongyrotropy in general couples the electromagnetic (transverse, left- and right-hand circularly polarized) and electrostatic (longitudinal) eigenmodes found in gyrotropic magnetoplasmas for parallel propagation. Removal of the nongyrotropic population recovers the three independent parallel propagating eigenmodes, and the associated dispersion equation, \(m_{++}m_{xx}m_{--} = 0\), contains the three classical dispersion equations for the left-hand circularly polarized electromagnetic (Alfvén/ion cyclotron) waves \((m_{++} = 0)\) for \((\omega, k)\), the right-hand circularly polarized electromagnetic (magnetosonic/whistler) waves \((m_{--} = 0)\) for \((\omega - 2\Omega_b, k)\), and the longitudinal electrostatic wave \((m_{xx} = 0)\) for \((\omega - \Omega_b, k)\).

The linear nongyrotropic dispersion equation depends only on the Fourier coefficients \(G_\sigma, G_{\pm 1}\), and \(G_{\pm 2}\) of the (complex Fourier series) expansion of the unperturbed nongyrotropic distribution \(F_{ob}\) which determine the characteristics of the ensuing coupling. Of particular interest are the situations with \(G_{\pm 1} = 0\) that are tantamount to \(\langle v_{\perp b o} \rangle = 0\), i.e. the unperturbed perpendicular current associated with the nongyrotropic species is null: the electrostatic mode decouples from the electromagnetic modes and only the interaction between the latter is possible. For the parallel propagation case under discussion, this type of gyrophase organization can bring about enhancements of (previously existing) electromagnetic instabilities with eventual modifications of their spectral characteristics. Needless
to say, nongyrotnopies with finite perpendicular currents provide richer effects. The (i) excitation of electrostatic and electromagnetic perturbations in media whose free energy sources are solely electromagnetic or electrostatic, and (ii) growth of electrostatic and electromagnetic waves in otherwise thoroughly stable (Maxwellian, isothermal, homogeneous) plasmas become then possible. Illustrations of these consequences of nongyrotnopy on the linear wave characteristics shall be provided below.

2.6 Physics of Nongyrotnopy

The analytical theory quoted above for parallel propagation in nongyrotnopic media shows the potential occurrence of coupling among the three polarizations [left-hand electromagnetic (+), longitudinal electrostatic (x), right-hand electromagnetic (−)] for spectral components exhibiting frequency shifts, respectively, of 0, Ωb and 2Ωb. We shall now clarify the underlying physics of interest, namely (i) why does the linear interaction take place among modes with different frequencies, (ii) how can transverse electromagnetic waves linearly couple to longitudinal electrostatic modes, and (iii) what is the interpretation of the solutions of the nongyrotnopic dispersion equation.

2.6.1 Frequencies of the coupled modes

The linear interaction among the modes is mediated by the nongyrotnopic particles. A necessary condition for an efficient coupling to take place is that the (fields of the) intervening modes apply forces on the nongyrotnopic particles that are perceived as having (approximately) the same frequency ("nongyrotnopic resonance"). Assuming for the moment that a linear interaction can indeed occur between longitudinal electrostatic waves and transverse electromagnetic modes (the next Section explains how this dialogue can take place), the particles of the nongyrotnopic species sense a force arising from the electrostatic wave with a frequency ωPx = ωx, where ωx denotes the wave frequency; similarly, but recalling that in the perpendicular plane these particles are rotating with their (signed) cyclotron frequency Ωb, they sense forces coming from the right-hand and left-hand circularly polarized electromagnetic waves with frequencies ωP+ = ω+ − Ωb and ωP− = ω− + Ωb, respectively.

In general, the gyrotnopic dispersion of the three polarizations precludes the (three-way) simultaneous satisfaction of the "nongyrotnopic resonance" condition for the three modes, ωPx = ωP+ = ωP−; as a rule, the coupling might occur between two modes at a time: ωPx = ωP+, or ωPx = ωP−, or ωP+ = ωP−. To visualize whether (any of) these necessary "resonance" conditions are satisfied, one can plot in the Brillouin plane the
dispersion curves of the gyrotrropic eigenmodes $\omega = \Omega_+(k)$, $\omega = \Omega_x(k)$, $\omega = \Omega_-(k)$, recalling that negative-frequency solutions of the $++$ dispersion equation are valid solutions associated with the $-(-)$ polarizations, and look for values of $k$ where the resonance conditions are satisfied, i.e., $\Omega_x(k) \approx \Omega_+(k) - \Omega_b$, or $\Omega_x(k) \approx \Omega_-(k) + \Omega_b$, or $\Omega_x(k) - \Omega_b \approx \Omega_-(k) + \Omega_b$. Clearly, the simplest way to conduct this search [keeping the convention adopted here of using the frequency of the left-hand mode as the unshifted frequency, $\omega = \Omega_+(k)$] is to plot in the Brillouin plane the dispersion curves of the (i) reference (unshifted) left-hand mode, $\omega = \Omega_+(k)$, (ii) electrostatic longitudinal mode upshifted by $\Omega_b$, $\omega = \Omega_x(k) + \Omega_b$, and (iii) right-hand mode upshifted by $2\Omega_b$, $\omega = \Omega_+(k) + 2\Omega_b$: "nongyrotrropic resonance" domains, i.e. potential interaction regions, exist when these sets of properly shifted dispersion curves intersect (or get close enough to) each other.

It is this physical need for the occurrence of "nongyrotrropic resonance" that brings about the analytical frequency shifts encountered in the matrix wave equation (4). Understandably, we expect the most intense nongyrotrropic effects (strongest deviations from gyrotrropy) to take place in the neighborhood of these resonances.

This reasoning is not affected by the frequency Doppler shifts $k \cdot v$ originated in the parallel velocities of the particles since the relevant intervening parallel waves, as shown in the wave matrix equation (4), share a common wavenumber: for a given particle the Doppler shifts would be identical.

2.6.2 Interaction between electrostatic and electromagnetic modes

The linear coupling between the two (left-hand and right-hand) circular polarizations brought about by nongyrotrropy does not require detailed justification. The associated transverse fields, rotating in opposite senses with angular velocities $\Omega_+$ and $\Omega_-$, act on the nongyrotrropic particles (whose perpendicular velocities rotate with angular velocity $\Omega_b$). If the gyrophase distribution of these particles contains second harmonic components ($G_2, G_{-2} \neq 0$), it is clear that the wave-particle interactions have conditions to resonate when $\Omega_+ \approx \Omega_- + 2\Omega_b$ is satisfied.

In contrast to the above transverse electromagnetic interaction, the eventual (linear, nongyrotrropic) coupling between transverse electromagnetic and longitudinal electrostatic modes is not so obvious. As already pointed out, this coupling hinges on the existence of a finite perpendicular current associated with the unperturbed nongyrotrropic species, i.e. $\langle v_{\perp b_0} \rangle \neq 0$.

Identifying with subscripts 0 and 1, respectively, the unperturbed and perturbed components of the number density $N_b$ and perpendicular velocity $v_{\perp b}$ of the nongyrotrropic population, the linearized perturbation of the
perpendicular current, $J_{\perp b1}$, which is a source of transverse (perpendicular) electromagnetic fields, becomes

$$J_{\perp b1} = q_b N_{bo} \langle v_{\perp b1} \rangle + q_b N_{b1} \langle v_{\perp bo} \rangle$$

(5)

where the angle brackets denote averages of $v_{\perp}$ taken over the corresponding (unperturbed and perturbed) velocity distributions. The perturbation density $N_{b1}$ associated with electrostatic modes can only drive transverse current densities (and hence, transverse electromagnetic modes) when $\langle v_{\perp bo} \rangle \neq 0$. The mean unperturbed perpendicular velocity of the nongyrotropic population is zero ($\langle v_{\perp bo} \rangle = 0$) when $G_{\pm1} = 0$.

Conversely, the electromagnetic modes can influence the electrostatic waves when the same condition is satisfied. Indeed, if $\langle v_{\perp bo} \rangle \neq 0$, the parallel component of the Lorentz force originated in the magnetic field of the electromagnetic modes, $q_b v_{\perp b} \times B_{emw}$, has a (linear) average nonzero value, $q_b \langle v_{\perp bo} \rangle \times B_{emw}$, that contributes to $\langle v_{|| b1} \rangle$ and, hence, to $J_{|| b1}$ and the longitudinal electrostatic field.

Strictly, the magnetic field generated by the unperturbed perpendicular current supported by the nongyrotropic population modifies the background magnetic field and invalidates the adopted solution of the unperturbed Vlasov equation. Because this perpendicular current, for identical gyrophase organizations, is proportional to the density and perpendicular thermal velocity of the nongyrotropic species, its effects on the (assumed homogeneous) ambient magnetic field can be roughly assessed by the perpendicular $\beta$ (ratio of plasma pressure to magnetic pressure) of the nongyrotropic particles.

2.6.3 Interpretation of the nongyrotropic dispersion

The nongyrotropic dispersion equation, $\text{Det} |m_{rs}| = 0$, obtained from the matrix wave equation (4), arises from the potential interaction of three parallel gyrotrropic wave modes [left-hand (+), longitudinal (x), and right-hand (−) polarizations] whose spectral components, as physically justified above, have been frequency shifted, respectively, by 0, $\Omega_b$ and $2\Omega_b$; its solutions reflect, in general, the coupling among these gyrotrropic modes that are no longer eigenmodes of the nongyrotropic system. Here we are concerned with the correct interpretation of the solutions $\omega(k)$ of the nongyrotropic dispersion equation, in order to properly assess the meaning of nongyrotropic Brillouin diagrams such as those shown, for example, in Figs. 5 and 10.

Let us consider one point ($\omega_1, k_1$) of a nongyrotropic dispersion curve, $\omega_1 = \Omega(k_1)$. Because this local dispersion stems from the coupling of (originally gyrotrropic) modes, it could contain, in general, contributions from the three interacting modes to the actual nongyrotropic field configuration.
Two questions arise at this point: (i) what are the relative weights of these contributions, and (ii) what are their frequencies in the reference frame adopted in the analysis. The answers follow directly from the previous discussion.

The wave matrix equation (4) defines the relative contributions of the three polarizations through the values of ratios such as $\frac{E_x(\omega_1 - \Omega_b, k_1)}{E_+(\omega_1, k_1)}$ and $\frac{E_-(\omega_1 - 2\Omega_b, k_1)}{E_+(\omega_1, k_1)}$: examples of this procedure are given in Brinca et al. (1992). As to the frequencies of these three wave components in the adopted reference frame, bearing in mind the physics of the frequency shifts discussed above, they are simply given as follows: the left-hand, longitudinal, and right-hand polarized contributions have frequencies, respectively, of $\omega_1$, $\omega_1 - \Omega_b$, and $\omega_1 - 2\Omega_b$.

In a nutshell, given a nongyrotropic solution $\omega(k)$, the matrix wave equation determines the relative contributions to the nongyrotropic configuration of the three polarizations at each point $(\omega_1, k_1)$: in general, the dispersion curve displays domains where only one, or two, of the three polarizations have contributions. For the contributing polarizations, their frequencies in the adopted reference frame ("real" frequencies) are obtained by undoing the frequency shifts introduced during the derivation of the dispersion equation (and, as we saw above, associated with the identification of regions with potentially strong coupling).

### 2.7 Illustration of Nongyrotropic Effects

In general, introduction of gyrophase organization introduces coupling among the eigenmodes of parallel propagation. However, if the perpendicular current supported by the unperturbed nongyrotropic population is zero, i.e. $\langle v_{\perp b o} \rangle = G_{\pm 1} = 0$, the longitudinal electrostatic mode cannot interact with the (right- and left-hand circularly polarized) transverse electromagnetic modes: only the latter may be coupled to each other. We shall illustrate situations with and without unperturbed nongyrotropic perpendicular current, resorting both to the numerical solution of the nongyrotropic dispersion equation and simulations with hybrid or particle codes. Gyrophase organization can be introduced in electrons or ions, and can assume arbitrary forms (only a few coefficients of their Fourier expansion enter the dispersion equation); however, for the sake of simplicity, the examples shown below usually adopt idealized gyrophase distributions.

#### 2.7.1 Zero unperturbed nongyrotropic perpendicular current

The effects on parallel wave stability for this nongyrotropic situation are illustrated with examples taken from Brinca et al. (1993a) that involve
ions with gyrophase organization. Similar effects originated in electron nongyrotropy can be found in Brinca et al. (1993b).

The medium is a hydrogen bi-maxwellian magnetoplasma with free energy provided by a proton temperature anisotropy \( A_p = 4 \) and other parameters as defined in Brinca et al. (1993a). The Brillouin diagram of the gyrotropic medium is shown in Fig. 4 and evidences an instability of the left-hand mode (\( \gamma > 0 \) corresponds to wave growth). Notice that the curve labeled \( m_{--}(\omega, k) = 0 \) results from the frequency upshift by \( 2\Omega_p \) of a negative-frequency branch of the right-hand (\( - \)) dispersion equation Brinca et al. (1993a), i.e. it is associated with the left-hand (\( + \)) polarization: in this case the coupling originated in the nongyrotropy only involves

![Figure 4: Part of the gyrotropic dispersion defined by \( m_{--} = m_{++} = 0 \). The proton temperature anisotropy drives the instability (from Fig. 2 of Brinca et al., 1993a).](image)
the left-hand wave, representing a modification of this mode.

We now introduce a simple, but extreme type of nongyrotropy: the initial gyrophases of the protons are forced to randomly assume the values 0 or \( \pi \), with the other parameters unchanged, clearly ensuring the inexistence of an unperturbed nongyrotropic perpendicular current. Figure 5 displays the numerical solution of the corresponding nongyrotropic dispersion equation. Two effects become apparent: the introduction of the gyrophase organization increases the maximum growth rate and shifts the unstable band to higher wavenumbers.

To verify these essential features, a simulation using a one-dimensional hybrid code with particle ions and massless fluid electrons is carried out; the

![Figure 5: Nongyrotropic dispersion obtained from the situation depicted in Fig. 4 through organization of the initial proton gyrophases that are allowed to assume only the values 0 or \( \pi \) (from Fig. 3 of Brinca, 1993a).](image)
calculations are quasi-neutral and thus only the coupling of electromagnetic modes occurs. The temporal evolution of the wave magnetic-field energies for the gyrotropic and nongyrotropic situations is depicted in Fig. 6; the faster growth brought about by the proton gyrophase organization is evident. Figure 7 shows the wavenumber spectra at the end of the linear growth interval in the two cases and, again, the analytical results are confirmed: it is clear the spectral shift towards higher wavenumbers in the nongyrotropic medium.

2.7.2 Finite unperturbed nongyrotropic perpendicular current

Examples taken from Motschmann and Glassmeier (1993) and Brinca et al. (1993b) illustrate the phenomenology for this nongyrotropic situation. The former investigation represents the first attempted interpretation of the Giotto G-S field observations possibly related to nongyrotropy, and introduces gyrophase organization in water vapor ions whereas the latter deals with electron nongyrotropy. In Brinca et al. (1992), the effects of nongyrotropic protons with finite perpendicular current are investigated
Figure 7: Wavenumber spectra obtained before saturation from the gyrotropic and nongyrotropic simulation runs (from the left panel of Fig. 5 of Brinca et al., 1993a).

analytically. Gurgiolo et al. (1993) study the generation of low and high frequency waves by gyrophase bunched ions at oblique shocks through hybrid 1D simulations; although the nongyrotropic particles carry a finite perpendicular current, the eventual interaction between electrostatic and electromagnetic modes is not considered here because the code is quasi-
The medium envisaged by Motschmann and Glassmeier (1993) is a cold hydrogen magnetoplasma permeated by a dilute (1% in number density) ring of newborn water vapor ions. The corresponding gyrotrropic dispersion, reproduced in Fig. 8, shows a wide spectrum of unstable wavenumbers. When the initial gyrophases of the newborn heavy ions is (extremely) organized, so that they all assume a constant value (complete alignment of their initial perpendicular velocities), the stability characteristics of the medium are modified, as indicated in Fig. 9, and the unstable wavenumber spectrum becomes narrow. This behavior might be relevant to the interpretation of highly coherent wave forms observed in the G-S cometary environment (Neubauer et al., 1993).

The situation chosen by Brinca et al. (1993b) to exemplify the linear coupling of electrostatic and electromagnetic modes with different frequencies evolves from a thoroughly passive (Maxwellian, isothermal, homogeneous) hydrogen magnetoplasma with very low \( \beta \) and \( \omega_{pe}/|\Omega_e| \approx 0.8 \), where \( \omega_{pe} \) denotes the electron plasma frequency: the gyrotrropic medium

![Graph](image)

Figure 8: Gyrotrropic dispersion (from Fig. 2 of Motschmann and Glaßmeier, 1993).
Figure 9: Nongyrotropic dispersion (from Fig. 3 of Motschmann and Glaßmeier, 1993).

has no free energy sources and is therefore stable. The gyrotropic parallel dispersion of the medium for frequencies of the order of $\Omega_e$ includes the electrostatic longitudinal Langmuir mode, the (right-hand circularly polarized) transverse electromagnetic whistler mode, and the (also transverse electromagnetic) right- and left-hand circularly polarized modes with cutoff frequencies symmetrically located ($\pm \Omega_e/2$) around $(\omega_{pe}^2 + \Omega_e^2/4)^{1/2}$ and phase velocities approaching $c$, as $k$ increases.

Adopting again a monochromatic gyrophase organization, this time for the electron population, the medium becomes unstable. Figure 10 shows the numerical solution of the nongyrotropic parallel dispersion equation in the neighborhood of an unstable domain brought about by the coupling of the Langmuir mode and the whistler wave (the real frequencies display the shifts incorporated in the matrix wave equation and already discussed). Observation of Fig. 3 of Brinca et al. (1993b) shows that the unstable coupling took place between the polarizations + and $x$, in the negative-frequency half-plane, i.e. between a right-hand (−) mode and a longitudinal electrostatic wave (negative-frequency solutions of the longi-
2.7. ILLUSTRATION OF NONGYROTROPIC EFFECTS

Figure 10: Nongyrotropic dispersion (from the upper panel of Fig. 4 of Brinca et al., 1993b).

tudinal dispersion equation are still associated with electrostatic waves). Recalling the discussion of Section 2.6, we realize that the (negative) frequency of the "whistler" mode can be read directly (because it arises from the + dispersion that was not frequency shifted), and that the frequency of the electrostatic contribution should be increased by $|\Omega_e|$ (since the $x$ polarization was "upshifted" by $\Omega_e$ which is negative).

To simulate this nongyrotropic situation involving the interaction of electrostatic and electromagnetic modes, Brinca et al. (1993b) utilize a one-dimensional particle code (a quasi-neutral hybrid code would not be able to properly describe the electrostatic fields even if the nongyrotropy were introduced in the protons and the relevant electrostatic mode were the ion acoustic wave). Figure 11 depicts the wavenumber spectra of the field components $E_x$ (electrostatic) and $B_z$ (electromagnetic) at three instants of time: as the simulation evolves, the spectra become increasingly more monochromatic. Figure 12 shows the temporal growth of the wave electric and magnetic field energies, and Fig. 13 displays the real dispersion of the same (as in Fig. 11) wave components $E_x$ and $B_z$. The simulation results are in close agreement with the numerical solutions of Fig. 10 (unstable wavenumber and growth rate amplitude); the different frequencies of the interacting modes confirm the interpretation made above of the frequency shifts occurring in the nongyrotropic matrix equation. The electrostatic mode has a frequency (normalized with respect to $|\Omega_e|$) $\omega_x \simeq 0.8$ and exerts a force on the electrons with the same frequency ($\omega_{pe} = \omega_x$); the right-hand whistler mode frequency is $|\omega_-| \simeq 0.2$ and thus acts on the
Figure 11: Wavenumber spectra obtained in the simulation of the nongyrotropic medium associated with Fig. 10 (from Fig. 5 of Brinca et al., 1993b).
Figure 12: Temporal evolution of the wave electric and magnetic energy densities obtained in the simulation cited in Fig. 11 (from Fig. 7 of Brinca et al., 1993b).

Figure 13: Real dispersion of electrostatic ($E_x$) and electromagnetic ($B_z$) wave components obtained in the simulation cited in Fig. 11 (from Fig. 6 of Brinca et al., 1993b).
electrons with a frequency \( \omega_{P_\perp} = 1 - |\omega_-| \approx 0.8 \), identical to \( \omega_{P_x} \), as required to have a strong interaction between the two (electrostatic and right-hand electromagnetic) modes.

2.8 Discussion

In hindsight, it seems appropriate to have divided the cometary era of \textit{in situ} wave observations and corresponding interpretations into \textit{before} and \textit{after} G-S. The data sets obtained at Giacobini-Zinner and Halley have motivated a wealth of research that has provided the first iteration in the interpretation of the experimental results. Although several outstanding puzzles persist, the overall picture points toward a reasonable understanding of the detected wave phenomenology. The more recent Giotto encounter with comet Grigg-Skjellerup occurred in an environment with different characteristics. Not only the gas production rate was the weakest of all the cometary encounters, but the intensity and orientation (with respect to the solar wind velocity) of the IMF were somewhat unusual: large amplitudes (Alfvenic Mach numbers lower than average) and \( \alpha \) values (well above the "average" \( \pi/4 \) angle) at 1 AU. These nonstandard solar wind conditions appear to have been responsible for some of the distinct characteristics of the G-S wave activity (viz. the observation of significant levels of left-hand waves) that can still be interpreted in the realm of the theories developed for G-Z and Halley. The novel aspect of the G-S observations underlined here, nongyrotropy, was brought about by the marked inhomogeneity of the environment (originated in the weak gas production rates) and seems to influence original features of the wave activity. Its interpretation shall require further developments in the theory of waves in nongyrotropic media.

Cometary wave activity thus continues to warrant intense research efforts. When inspired by observations carried out before the G-S era, they shall probably be centered on the nonlinear domain (e.g. nonlinear evolution equations applied to the description of discrete, coherent wave events) and will encompass, for the post-G-S epoch, both linear (e.g. oblique propagation, inhomogeneity effects) and nonlinear studies of wave properties influenced by particle populations with gyrophase organization.

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