PARTICLE BEHAVIOR IN THE MAGNETOSPHERE

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ABSTRACT

The Rice Convection Model deals with large-scale processes in the Earth's inner and middle magnetosphere, including coupling to the ionosphere. Starting from appropriate initial and boundary conditions, the model computes the following physical parameters: ionospheric electric fields and currents; magnetospheric particle distributions, electric fields, and electric currents; and magnetic-field-aligned (Birkeland) currents connecting the two regions. This paper reviews work on the model with emphasis on the assumptions made, the basic equations, and the numerical methods. The theoretical basis of the model is compared and contrasted with standard magnetohydrodynamics. The limitations imposed by the major assumptions are discussed. Model inputs and boundary conditions are listed, and the methods of specifying them discussed. Some physical conclusions and insights that have been gained from the model are listed and described very briefly. References are given to published discussions of the major points of physics.

1. INTRODUCTION

The Rice Convection Model (RCM) deals with the closed-magnetic-field-line region of the inner magnetosphere, including the coupling of the inner magnetosphere to the ionosphere. The model is based on the "quasi-static" or "slow-flow" approximation, in which magnetospheric particles are assumed to be drifting slowly relative to their thermal velocities. The requirement that the electric current density be divergence-free leads to the flow of Birkeland currents along magnetic field lines, connecting magnetospheric and ionospheric current patterns. The divergence of magnetospheric drift current is balanced by the divergence of ionospheric conduction current. The

model keeps track of electric fields and currents in both the ionosphere and magnetosphere, as well as the time evolution of the magnetospheric particle population. The magnetic field model is taken as input, not computed self-consistently with the model currents. Use of an input magnetic field model reduces a three-dimensional problem to two coupled two-dimensional problems.

![Diagram of the Earth's magnetosphere](image)

**Figure 1.** View of the Earth's magnetosphere in the noon-midnight meridian plane. The Sun is to the left. Hollow arrows indicated flow velocities.

The RCM is a quantitative formulation of standard magnetospheric convection theory. The basic idea of magnetospheric convection is illustrated in Figure 1. The flow of the solar wind past the magnetosphere causes plasma in the outermost part of the magnetosphere to flow systematically away from the Sun. There is a sunward return flow in the interior of the magnetosphere. Because magnetic field lines generally tend to be good conductors, this systematic magnetospheric circulation pattern causes a related flow pattern in the ionosphere: antisunward over the polar caps, which connect to the lobes of the magnetotail, and sunward through the lower-latitude part of the auroral zone, which maps to the plasma sheet.

Of the various types of numerical simulations now being applied in space plasma physics, the global MHD simulations discussed in this volume by and Wu (1984), and in various earlier papers (e.g., LeBoeuf
et al., 1978; Lyon et al., 1980; Brecht et al., 1981; Wu et al., 1981) most resemble the RCM. Like the global MHD simulations, the RCM is used to investigate large-scale phenomena in the Earth's magnetosphere. However, there are many differences between these global MHD models and the RCM, in origin, in approach, and in physical approximations used.

The RCM does not represent a conversion to space physics of a code developed for controlled-fusion or weapons research. Instead, the RCM is a one-group effort at a very specialized code. Our work has its roots in earlier qualitative and analytic theory, which began with Axford and Hines (1961), Dungey (1961), and Cole (1961). It was developed and made more quantitative by Fejer (1964), Taylor and Perkins (1971) and particularly by Vasyliunas (1970, 1972). In the seventies, emphasis gradually shifted to computer models (Wolf, 1970; Swift, 1971; Jaggi and Wolf, 1973), because it appeared that analytic calculations were too limited to solve the system of equations for reasonably realistic conditions.

Another distinguishing feature of the RCM is the extensive boundary condition requirements, especially as compared to the global MHD models. Aside from the initial-condition information, the MHD models require only very simple physical input data, namely ρ, T, B, v of the solar wind. In addition to similar initial-condition information, the RCM also needs the following: potential distribution on the polar cap boundary; plasma sheet n_e, T_e, and T_i at the model's outer boundary; a global ionospheric conductivity model; and a three-dimensional magnetic field model. One result of the heavy reliance on boundary conditions is to tie the model strongly to data. We frequently model specific well-observed events. We use some data from the event as input; other data are compared with a wide variety of model predictions. These frequent direct theory-vs.-observation comparisons sometimes yield important hints about the physics, and generally help to keep us in touch with reality.

A second result of the heavy use of boundary conditions is that we do a lot of computer experiments. The model has many knobs to turn in order to probe the physics of the system—to determine what causes what. Thus the heavy reliance on boundary conditions, which initially seems like a disadvantage of our approach compared to the global MHD models, turns out actually to be an advantage, in many cases.

Section 2 of this paper states and discusses the limiting assumptions of the model, and gives the basic equations. Section 3 describes the inputs to the model, and how we specify them. The overall logic of the program and the numerical methods used are described in Section 4. Section 5 lists what we regard as the major physical conclusions and insights gained from the modeling effort, and gives references to published discussions of them.
2. BASIC EQUATIONS AND PHYSICAL ASSUMPTIONS

Basic Equations

First consider the equation for bounce-averaged adiabatic drift of particles bouncing on closed field lines. We assume that the kinetic energy associated with particle drift is small compared to the particle's thermal motion (slow-flow approximation). Consequently, we can express the particle's kinetic energy in the form

\[ E_K = E_K(x_e, \mu, J) \]  

(1)

where \( x_e \) = equatorial crossing point and \( \mu \) and \( J \) are the first two adiabatic invariants. Then the equation for bounce-averaged drift velocity becomes

\[ \dot{\mathbf{v}}_e = \frac{B_e(x_e) \times \mathbf{v}_e E_K(x_e, \mu, J)}{qB_e^2} + \frac{E(x_e) \times B_e(x_e)}{B_e(x_e)^2} \]  

(2)

where the first term represents gradient/curvature drift and the second is \( \mathbf{E} \times \mathbf{B} \) drift. Subscript "e" refers to the equatorial plane. (For a derivation of the first term, see Northrop [1963] or Wolf [1983]..) Instead of considering particles of each pitch angle separately, we usually take the pitch-angle distribution to be isotropic, for the sake of simplicity. We effectively assume that the particles' pitch-angles are scattered in a time short compared to the convection time. Particle energies are assumed not to change in these scattering processes. The particle's kinetic energy is then given by

\[ E_K = \lambda S \]  

(3a)

where

\[ S = \left( \int ds/B \right)^{2/3} \]  

(3b)

and \( \lambda \) is called the energy invariant. The integral \( \int ds/B \) extends along a field line from the southern ionosphere to the northern and represents the volume of a tube of unit magnetic flux. The corresponding total drift velocity is given by

\[ \dot{\mathbf{v}}_e = \frac{\lambda B_e \times \mathbf{v}_e S}{qB_e^2} + \frac{E_e \times B_e}{B_e} \]  

(4)

(Harel et al., 1981a; Wolf, 1983).

Such a collection of adiabatically drifting particles satisfies a modified frozen-in flux theorem. Namely, consider particles drifting according to a law of the form

\[ \dot{\mathbf{v}}_e(x_e) = \frac{E_e \times B_e}{B_e^2} + \frac{\mathbf{v}_{e} Y \times B_e}{B_e^2} \]  

(5)
Let subscript "s" refer to a given particle species, specifically a particle of a given charge and given values of the relevant invariant(s). Let \( \eta_s \) = number of particles per unit magnetic flux for particles of species \( s \). Then, neglecting loss, the continuity equation in the equatorial plane can be written

\[
\frac{\partial (\eta_s B_e)}{\partial t} + \nabla_e \cdot (\eta_s B_e \nabla_e) = 0
\]  
(6)

with equation (5) and Faraday's law, equation (6) can be simplified to the form

\[
\left( \frac{\partial}{\partial t} + \nabla_e \cdot \nabla_e \right) \eta_s(x_e, t) = 0
\]  
(7)

In other words, neglecting loss, the number of particles of a given species, per unit magnetic flux, is constant along a drift path.

Currents play a central role in our formulation of magnetospheric convection. The essential physics coupling the inner magnetosphere and ionosphere comes from equating the divergence of magnetospheric drift current, appropriately scaled, to minus the divergence of ionospheric conduction current. Consider the current carried by particles drifting in the magnetosphere. It is convenient to map these drift currents to the equatorial plane, and to discuss the current flowing in that plane per unit length perpendicular to the current. In this formalism the current is the sum of the magnetization current and the gradient/curvature-drift current. Using (4), we obtain

\[
\mathbf{j}_e(x_e) = \text{magnetization current} + \sum_S \eta_s(x_e) \mathbf{l}_S \mathbf{b}_e \times \nabla_e S
\]  
(8)

where \( \mathbf{b}_e \) is a northward unit vector parallel to \( B_e \). Taking the divergence (which makes the magnetization-current term vanish), we obtain

\[
\nabla_e \cdot \mathbf{j}_e(x_e) = - \mathbf{b}_e \cdot \nabla_e \left[ \sum_S \mathbf{l}_S \right] \times \nabla_e S
\]  
(9)

Since the pressure is given by

\[
p = \frac{2}{3} \sum_S \eta_s \mathbf{l}_S \mathbf{S}^{5/2}
\]  
(10)

Equation (9) becomes

\[
\nabla_e \cdot \mathbf{j}_e(x_e) = \mathbf{b}_e \cdot \nabla_e p(x_e) \times \nabla_e \left[ f ds/B \right]
\]  
(11a)

\[
= \frac{2}{3} \sum_S \mathbf{l}_S \mathbf{S}^{5/2} \mathbf{b}_e \cdot \nabla_e \eta_s \times \nabla_e \left[ f ds/B \right]
\]  
(11b)

\[
= -2 J_{\parallel e}
\]  
(11c)
where $J_{\parallel e}$ is the density of Birkeland current up from the ionosphere, mapped to the equatorial plane. The factor of 2 accounts for there being two ionospheric ends of the field line, one in the northern hemisphere, one in the southern (assumed to have equal conductivities for simplicity). Equation (10) can actually be applied anywhere on the field line, including at the ionospheric ends. One can calculate the density of Birkeland current down into the ionosphere by taking gradients in $p$ and $\int ds/B$ just above the ionosphere.

The same equation can be derived from the momentum equation of MHD, namely

$$\rho \frac{Dv}{Dt} = -\nabla p + \frac{J}{\rho} \times B$$  \hspace{1cm} (12)

Crossing with $B$ and solving for the component of $J$ perpendicular to $B$ gives

$$J_{\perp} = \frac{B \times \nabla p}{B^2} + \frac{B \times \left( \frac{Dv}{Dt} \right)}{B^2}$$  \hspace{1cm} (13)

Within the slow-flow approximation, we can neglect the last term on the right side of (13). The condition that $\nabla \cdot J = 0$ can be written

$$B \frac{d}{ds} \left( \frac{J_{\perp}}{B} \right) + \nabla \cdot J_{\perp} = 0$$  \hspace{1cm} (14)

Integrating along the field line, and performing several vector-calculus manipulations, we obtain

$$\frac{J_{\parallel}}{B_1} = \frac{J_{\parallel e}}{B_e} = -\frac{1}{2B_e} \nabla \times V \times V \left[ \int ds/B \right]$$  \hspace{1cm} (15)

This equation, which is the same as we obtained above from drift theory (eqn. 11), was derived by Vasyliunas (1970).

For the case where there is no neutral wind, the equation for height-integrated horizontal ionospheric current can be written

$$J = \frac{1}{\Sigma \omega} (\nabla_V V)$$  \hspace{1cm} (16)

where $\Sigma_V$ represents the horizontal gradient operator in the ionosphere, and

$$\Sigma_{XX} = \int \sigma_1 dh / \sin^2 \lambda$$
$$\Sigma_{XY} = -\Sigma_{XY} = \int \sigma_2 dh / \sin \lambda$$
$$\Sigma_{YY} = \int \sigma_1 dh$$  \hspace{1cm} (17)

Here $x$ and $y$ are in the magnetic northward and eastward directions, respectively, $\lambda$ is the magnetic dip angle, and $\sigma_1$ and $\sigma_2$ are the local Pedersen and Hall conductivities, respectively. These simplified forms for the height-integrated conductivities are valid for all of the ionosphere, except very near the dip equator. The general
equations were derived by, for example, Fejer (1953). Equating the divergence of ionospheric current in (16) to the field-aligned current from (15) yields the fundamental equation for large-scale magnetosphere-ionosphere coupling:

\[ J_{\parallel \sin I} = \frac{B_{\parallel \sin I}}{2B_e} \cdot \hat{z} \cdot \nabla_e \times \nabla_e \left[ \int ds/B \right] = \nabla_h \cdot \left( \frac{\Sigma}{\infty} \cdot (-\nabla_h \nabla) \right) \] (18)

where \( \hat{z} \) is a northward unit vector perpendicular to the equatorial plane. (This equation was originally derived by Vasyliunas (1970).)

Table 1 compares our convection-model approach to standard ideal-fluid MHD, as follows:

1. The continuity equation exists in both formalisms. However, in our approach, it is expressed in terms of \( n_s \), the number of particles of species \( s \) per unit magnetic flux. Also, in the inner magnetosphere, gradient/curvature drifts are typically comparable to \( E \times B \) drifts, so that different particle species drift at different velocities. Consequently, a multifluid continuity equation is needed.

2. If we neglect the inertial term in the MHD momentum equation, its three components reduce to the condition that \( p \) is constant along a field line and equation (13) for the component of \( J \) perpendicular to \( B \).

3. We apply the adiabatic-expansion form of the MHD energy equation, with two modifications: (i) we apply it to each fluid species individually, and (ii) we apply it to an entire flux tube at once, using the fact that pressure and density are constant along a magnetic field line.

4. The input magnetic field models have \( \nabla \cdot B = 0 \).

5. The Ampere's law Maxwell equation is not satisfied in our model, in the sense that \( B \) is not forced to be consistent with our computed \( J \).

6. The Faraday's law Maxwell equation is included implicitly in our simulations. Because the input magnetic field model changes with time, the equatorial mapping point of a given point in the ionosphere changes with time. This motion is interpreted as \( E \times B \) drift in an induction electric field.

7. The usual ideal-MHD perfect-conductor assumption is adopted, with regard to the component parallel to \( B \). The components of \( E + \gamma \times B = 0 \) perpendicular to \( B \) are modified to allow the different plasma species to gradient/curvature drift perpendicular to \( B \), in addition to the \( E \times B \) drift.
TABLE I. Comparison of Ideal-Fluid MHD with Conveciton Model

<table>
<thead>
<tr>
<th>Ideal-Fluid MHD</th>
<th>Convection Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$</td>
<td>$\frac{D \eta}{Dt} = 0$</td>
</tr>
<tr>
<td>2. $\rho \frac{\partial \mathbf{v}}{\partial t} - \nabla p + J \times \mathbf{B}$</td>
<td>$\mathbf{j}_1 = \frac{\mathbf{B} \times \mathbf{V} \sum p_S}{B^2}, p_S \text{ is constant on field line}$</td>
</tr>
<tr>
<td>3. $\frac{D}{Dt}(\rho v^{-5/3}) = 0$</td>
<td>$p_S \int ds/B^{5/3} = 2\rho \eta_S/3$</td>
</tr>
<tr>
<td>4. $\mathbf{v} \cdot \mathbf{B} = 0$</td>
<td>$\mathbf{v} \cdot \mathbf{B} = 0$ in input $\eta_S$ model</td>
</tr>
<tr>
<td>5. $\mathbf{v} \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \mathbf{E}_0 / \mathbf{a}_T$</td>
<td>Not included self-consistently</td>
</tr>
<tr>
<td>6. $\mathbf{v} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$</td>
<td>Included implicitly in time-dependent mapping</td>
</tr>
<tr>
<td>7. $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$</td>
<td>$\mathbf{E} \cdot \mathbf{B} = 0$</td>
</tr>
<tr>
<td></td>
<td>$\mathbf{v}_S = \mathbf{v}_S + \mathbf{E} \times \mathbf{B}$ drift + (grad/curv. drift)$_S$</td>
</tr>
</tbody>
</table>

Discussion of Assumptions

We now return to discuss the physical implications of our principal assumptions.

Slow-Flow (Quasi-Static) Approximation. Our neglect of inertial drifts compared to gradient and curvature drifts is equivalent to neglecting the inertial term in the MHD momentum equation (13). This approximation is valid if the plasma flow velocity is small compared to the sound speed, and the time scale for variations is long compared to typical wave travel times. It is one of our principal limitations compared to MHD. This approximation limits our ability to treat substorms in detail, because some substorm phenomena occur on very short time scales (less than a minute). Also, flow velocities close to the sound speed have sometimes been observed even in the innermost parts of the plasma sheet during substorms (Moore et al., 1981). We do not treat MHD waves — fast, intermediate, or slow — in our formalism. The neglect of the inertial terms is a non-trivial limitation, even for steady-flow situations. If we combine standard empirical magnetic field information (e.g., Behannon, 1970) with standard estimates of the cross-tail potential drop, we would estimate that the velocity of earthward convection exceeds the sound speed.
somewhere around lunar orbit (60 $R_E$ geocentric distance). Thus our model could not, even for steady conditions, be applied out as far as lunar orbit. The model also cannot be applied close to an X-type neutral line, since plasma normally flows away from such a line at a speed that exceeds the Alfvén speed in the external plasma, and the Alfvén speed is of the same order as the local sound speed. Nevertheless, the model nearly always can be safely applied to phenomena that occur in the inner magnetosphere on time scales greater than a few minutes.

**Magnetic Field Model Taken as Input.** Historically, the theory of inner-magnetospheric convection was formulated (e.g., Vasyliunas, 1970; Wolf, 1970) assuming that the magnetic field was known, because, at the time, the magnetic-field configuration of the magnetosphere was much better known than the electric field, which was the quantity to be calculated. There were several useful and well-established semi-empirical magnetic-field models (e.g., Mead and Beard, 1964) available in card-deck form for scientific use. In contrast, only a few measurements of auroral-zone electric fields had been made (e.g., Mozer and Serlin, 1969), and there was still substantial debate over whether magnetospheric convection actually existed. This situation has now changed somewhat, due mainly to extensive electric field measurements made by polar-orbiting spacecraft and incoherent-backscatter radars. The RCM regards the magnetic field as known input for "practical" reasons. To calculate the magnetic field self-consistently would enormously increase the size and complexity of our calculation. It would change the present system of two coupled two-dimensional problems (one in the ionosphere, one in the magnetospheric equatorial plane) to a fully three-dimensional calculation. We avoid doing a full three-dimensional self-consistent calculation by using the magnetic field model to link the ionospheric and magnetospheric calculations, and to map magnetospheric pressure, etc., along field lines. Along with the slow-flow approximation, the use of an input magnetic-field model limits our model’s region of applicability to the inner magnetosphere and near tail. Further out in the tail, the beta of the plasma is very large, and the relevant characteristics of the magnetic field structure are too highly variable to be reliably deduced from available magnetic-field models. G.-H. Voigt and Gary Erickson in our group are currently working on the development of magnetic field models that are consistent with convection. It is a difficult problem, in terms of both numerical analysis and physics.

The global MHD models do not share our difficulties with self-consistently modeling $B$ and convection. Their powerful computational machinery automatically computes magnetic-field configurations that satisfy both Ampere’s law and the momentum equation. However, at present these models cannot effectively address the interesting physics of the inner magnetosphere and near tail regions ($x = -4 R_E$ to $-20 R_E$). The MHD models are limited in this respect because (i) they necessarily employ large grid spacing, (ii) economics prevents them from modeling for particle-drift time scales (typically several hours), (iii) particle transport by gradient and curvature drift
(which they neglect) plays a major role in the physics of the region, and (iv) the ionosphere, which also plays an important role, is very difficult to include in a global-scale MHD model.

Thus we continue to pursue the quasi-static approach, despite the difficulties involved. It continues to be the best way to model the inner magnetosphere and large-scale magnetosphere-ionosphere coupling. The quasi-static approach (slow-flow approximation) is, of course, only applicable near the Earth. Unlike the global-MHD models, it cannot be used legitimately to model the far tail or solar-wind/magnetosphere coupling. However, it is presently the best way to treat most large-distance-scale, long-time-scale phenomena in the inner and middle magnetosphere.

3. MODEL INPUTS

We now discuss the various input models needed for the main simulations. It is here that one can sense the strains and sacrifices involved in trying to realistically model actual events. Much of the detailed disagreement between model results and observations is undoubtedly due to inaccurate input.

Potential Distribution on the Poleward Boundary of the Model

The potential distribution on our poleward boundary is needed as a boundary condition, so that the elliptic equation (18) can be solved for V.

First consider the total potential drop across the boundary. This parameter, which measures the total strength of convective flow through the model system, is approximately equal to what observers call the "polar cap potential drop." We have considered three methods for estimating this crucial time-dependent parameter from observations. The most obvious and direct source is the electric field measured by a polar-orbiting spacecraft in dawn-dusk orbit. We used such data for our simulations of the substorm of September 19, 1976, but, unfortunately, such direct measurements are not usually available with sufficient frequency to be helpful for event simulations. A second approach, which is applicable to many more events, is to estimate the polar-cap potential drop from the observed interplanetary magnetic field and an empirical formula (see, e.g., Reiff et al., 1981). This basic approach was used for our simulations of the magnetic storms of July 29, 1977 (Wolf et al., 1982) and March 22, 1979 (Spiro and Wolf, 1983). (See top panel of Figure 2.) Another possible approach involves the use of a magnetogram-inversion scheme (Kamide et al., 1981). Given a conductivity model and a large number of ground magnetograms as input for a specific event, this scheme computes global electric current and electric-field patterns, and consequently, the polar-cap potential drop. Reiff et al. (1983) report remarkably good agreement between polar-cap potential drops.
inferred by the magnetogram-inversion procedure for the March 22, 1979 magnetic storm and values predicted by using IMF data and an empirical formula for the same event.

![Graph of Vdrop (kV), AE, and standoff distance over time]

**Figure 2.** Time-dependent input parameters for simulation of the March 22, 1979 magnetic storm. The top panel shows the polar-cap potential drop estimated from solar-wind parameters (P. H. Reiff, private communication, 1982). The middle panel shows the auroral-electrojet index, which we need to choose the conductivity model. The bottom panel shows the magnetopause standoff distance, as estimated from the solar-wind ram pressure (G.-H. Voigt, private communication, 1982).

With regard to the local-time variation of the potential, the classic pattern of magnetospheric convection corresponds to a maximum potential on the dawn side, a minimum on the dusk side. This established local-time variation is expressed very roughly by the simple equation

$$V = -V_0 \sin \phi$$  \hspace{1cm} (19)

where $V_0$ is a constant, and $\phi$ is the local-time angle ($=0$ at local noon, $\pi/2$ at dusk, etc.). Although there is some observational information on deviations from this simple law (see, e.g., Foster, 1983), there is very little systematic information on how the local-time variation depends on magnetic activity. However, substantial evidence of complex, non-classical convection patterns during periods
when the interplanetary magnetic field is northward for a prolonged period have been presented (see, e.g., Burke et al., 1979). Of course, $V_0$ is relatively small under these conditions anyway, and flows tend to be weak.

**Ionospheric Conductances**

The daytime ionospheric conductance (height-integrated conductivity) depends on the strength of ionizing solar UV radiation, which, in turn, depends on sunspot cycle. Sufficient data exist from the various incoherent backscatter radars around the world to provide adequate observational information on the sunspot-cycle dependence of

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**Figure 3.** Ionospheric Pedersen and Hall conductances for quiet and active times, as used in our simulation of the March 22, 1979 event. The distributions are based on the empirical electron-flux model of Spiro et al. (1982), but the distributions are adjusted in latitude according to the computed electron inner edges. The "active time" diagrams pertain to an AE index near 1000.
ionospheric conductance. Unfortunately, no one has systematically collected and compiled the information, and consequently, there are possible errors of nearly a factor of two in our best conductance estimates in the sunlight-dominated region.

Auroral electron precipitation has a major effect on ionospheric conductance at high latitudes. To estimate the effect of auroral precipitation, we have used parameterized results of ionospheric models to estimate the conductance corresponding to a given auroral electron flux (see Spiro et al. [1982] for detailed discussion and references). To estimate the global distribution of auroral electron fluxes as a function of time during the event, we have used two approaches. One approach is to use electron data from polar-orbiting spacecraft for the event in question. The second approach is to utilize a statistical study of auroral electron fluxes, sorted according to a magnetic index (the AE and AL indices seem most useful). Sample conductances based on such a statistical study are shown in Figure 3. We then combine the AE or AL index measured as function of time through the event with the statistical model to arrive at a global, time-dependent conductance model. The second panel of Figure 2 shows the idealized version of the preliminary AE index that was used to generate the conductance models for the March 22, 1979 event.

Both of these approaches to obtaining global conductance models have problems. The real-time spacecraft measurements should give accurate conductivities along the orbit track, but major interpolations and extrapolations are required to construct global models for all times during an event. Statistical models generally give smooth electron-flux distributions that do not accurately portray the physical situation at a given time. An AE index of 100 may correspond to the peak of a small, isolated substorm occurring during a long quiet period, or it may correspond to a lull between successive substorms in a large magnetic storm. The electron flux distributions will be quite different in these two situations, but the statistical analysis averages them together. For a comparison of statistical-average conductances with those derived from spacecraft overflight, see the paper by Reiff (1983) or Simons et al. (1983).

Magnetic Field Models

For our recent simulations, we have used time sequences of Voigt (1981) magnetic field models. For each model in the sequence, the magnetopause standoff distance is adjusted to correspond to the solar-wind \( v_w \) observed at that time (see middle panel of Fig.2), and the ring-current strength is adjusted according to the observed Dst index. The location of the ring current, and the strength and location of the "neutral sheet current" are sometimes adjusted to agree with magnetic fields or particles observed by spacecraft at the time. Analogous sequences of models were previously constructed by Olson and Pfister (1982) for the July 29, 1977 magnetic storm.
Hot-Particle Distribution at our Outer (High-L) Boundary

In most cases, we have somewhat arbitrarily set our plasma-sheet boundary condition to correspond to nominal values of plasma-sheet parameters. In the case of the March 22, 1979 event, the ISEE-2 spacecraft was in the plasma sheet near our tailward boundary for most of the event, so that we could adopt a more realistic plasma boundary condition. Of course, single-satellite measurements give no information on the variation of plasma-sheet parameters along the boundary.

Initial Hot-Particle Distribution

As an initial condition, we specify the particle distribution function throughout the magnetosphere. This initial distribution is not very important for the plasma sheet, because the particles that initially populated the plasma sheet are soon replaced by others. However, the distribution of particles trapped near the Earth is important, because many of the particles that were initially on trapped orbits remain on trapped orbits throughout the event. In specifying the initial plasma condition, we have generally made reasonable guesses representing average conditions plus whatever fragmentary information was available for the specific day in question.

Other Possible Inputs

Non-zero neutral-wind velocity in the current-carrying layers of the ionosphere affects the ionospheric current-voltage relation (16). To specify the neutral-wind correction term for (16), we actually need a complete wind model — three-dimensional and time — for the E and F layers of the ionosphere. Observational summaries are not nearly that comprehensive, but reasonable three-dimensional theoretical neutral-wind models are now available (e.g., Roble et al., 1982). We expect to examine neutral-wind effects in the next few years. In models constructed so far, however, we have arbitrarily set the neutral-wind velocity equal to zero.

Given a correct algorithm for computing the magnetic-field-aligned potential drop on each field line, we could relatively easily incorporate these potential drops in our model. One simple, reasonable approach would be to use a statistical study of particle data (e.g., Yeh and Hill, 1981) to infer the distribution of field-aligned potential drops, and we hope to do that soon. In the meantime, we assume that there is no electric field parallel to the magnetic field.

Particle loss is another physical process that we have been neglecting, for simplicity. We do not know of any simple algorithm that would allow us to predict either ion or electron precipitation correctly. The assumption of strong pitch-angle scattering, with a full loss cone, is an optimally simple assumption (see, e.g., Kennel
(1969) for a discussion. It would tend to give an upper estimate of the actual precipitation rate. Electron precipitation can result in substantial loss of electrons from a flux tube in less than a typical convection time. However, the effect on the model is not enormous, because electrons contribute a small fraction of the total pressure in the plasma sheet and ring current. It should be noted that the most important effect of electron precipitation on the system, namely the effect on the conductivity of the ionosphere, is already taken into account, in a rough, empirical way, in the model of ionospheric conductances. Ions are lost by both precipitation and charge exchange. The latter process is a bit more predictable since it depends directly on atomic reaction rates and a neutral-atmosphere model, less directly on subtle plasma physics. We anticipate, in the next few years, including charge-exchange loss in the model.

Intense fluxes of kilovolt ions have been observed streaming up out of the ionosphere (Shelley et al., 1976). These fluxes seem to be associated with upward Birkeland currents. Correspondingly, the ionospheric ion species O⁺ seems to contribute about half of the ring current energy, at least at energies below about 20 keV (see, e.g., Balsiger, 1983). Observational information is rapidly approaching the point where a reasonable observation-based algorithm could be formulated for estimating the upward ion fluxes. We anticipate incorporating such an empirical model of ionospheric injection in our model.

The present version of our model defines an idealized inner magnetosphere, with no neutral winds, no field-aligned potential drops, no particle loss from the plasma sheet and ring current, and no ion injection directly from the ionosphere to the magnetosphere. However, even with all these interesting and uncertain points of physics out, there is a lot of physics to investigate. We are still doing computer experiments to determine how all of the various input parameters in our simplified, idealized magnetosphere — the potential on the polar-cap boundary, the distribution of ionospheric conductance, the magnetic-field model, the plasma distribution at our outer boundary, our initial plasma distribution — affect the various calculated observable quantities (e.g., ionospheric currents, Birkeland currents, ionospheric electric fields, ring current strength, ring current location). These are very complicated functional relationships. After several years of work on the problem, we feel that we are only now nearing the point where we have sorted out most of the dominant causal connections. When we get to that point, we will begin adding new physical processes (field-aligned potential drops, etc.) and investigating their impact on this complicated system. We will add them one at a time, doing enough computer experiments on each to try to sort out the dominant cause-and-effect relationships.
4. PROGRAM LOGIC AND NUMERICAL METHODS

Program Logic

Figure 4 shows the essential logic of our program. We start at the top of the diagram and proceed counterclockwise. The initial magnetospheric plasma configuration is specified, as is the magnetic-field model for the initial time $t_0$. The magnetospheric current density can then be calculated from equation (8), and the density of Birkeland current is computed by taking the appropriate divergence (equation 11). Assuming no neutral wind, and with specified models for the ionospheric conductance and polar-boundary potential at time $t_0$, equation (18) is solved for the ionospheric potential.

Figure 4. Basic logic diagram of our main program, an elaboration of a diagram published by Vasyliunas (1970). The program goes once around the diagram each time step. Rectangles represent primary computed quantities. Rounded boxes signify input information. Arrows with dashed lines indicate physics that could straightforwardly be included in the model, but has not been yet.

distribution. The second boundary condition on (18) is that there be no current across the equator; in practice, we actually require zero current across a low-latitude boundary at about 20° latitude. Taking the field lines to be equipotentials, we transform the potential to an "inertial" frame (not rotating with the Earth) using the equation
\[ V_m = V - (92,400 \text{ volts}) \sin^2 \theta \]  

(20)

where \( \theta \) = ionospheric colatitude. We then map this potential distribution out to the magnetospheric equatorial plane. The positions of magnetospheric particles are then advanced according to the law

\[ x_e(t_0 + \Delta t) = x_e(t_0) + \left( \frac{B_e \times V_e V_m}{B_e^2} + v_{GC} + v_g \right) \Delta t \]  

(21)

where \( \Delta t \) is the time step, \( V_m \) is the potential mapped up from the ionosphere, \( v_{GC} \) is the bounce-averaged gradient/curlature drift velocity, and \( v_g \Delta t \) represents the local motion of our equatorial grid between times \( t_0 \) and \( t_0 + \Delta t \). (Our grid is fixed in the ionosphere. Our equatorial grid represents the equatorial map of the ionospheric grid, but it changes in time if the magnetic field model changes.) The \( v_g \Delta t \) term in equation (21) can thus be regarded as \( E \times B \) drift in an induction electric field. Neglecting particle loss or addition, we compute the new particle distribution for time \( t_0 + \Delta t \) using equation (7) with the total effective velocity from equation (21). The whole procedure is then repeated each time step.

Numerical Methods

At the time that our basic numerical method was developed (1975-1976), computing charge rates were sufficiently high that we could only afford to carry out the calculations for a coarse grid. Thus, the numerical procedure was selected for use on a grid with a relatively small number of grid points (~600). Since computing costs have dropped drastically since 1975, and core sizes have increased dramatically, it is now economically feasible to run the program with a much denser grid, for increased detail and accuracy. We are in the process of writing a more sophisticated program, using more elaborate numerical methods for efficient handling of the system with a denser grid. However, that work is not complete yet, and we will describe here the old numerical methods.

Representation of the Particle Distribution. We consider an isotropic plasma and represent the energy distribution in terms of a finite number of "energy channels." Each channel actually corresponds to a given value of the energy invariant \( I \) (equation 3) and given charge \( q \). In most of the simulations done so far, we have used 21 of these channels. Representing the plasma simply by densities at grid points seemed to offer inadequate spatial resolution, given the number of grid points that we could afford. Therefore, we decided to represent the particle distribution in terms of computed inner edges, not in terms of densities at grid points. For a given energy channel, a sharp density jump tends to form physically at the inner edge of the plasma sheet at the natural boundary between the region filled with fresh plasma that has flowed in from the tail, and trapped particles that have been circling the Earth for an extended period. The plasma pressure changes drastically over one grid space at this inner edge.
which is just the region we wish to model carefully. In our most recent simulations, we have been assigning each invariant-energy level several different inner edges, each one corresponding to a different level of invariant density. In other words, we represent the distribution of particles with a given \( l \) in terms of directly computed contours of constant \( \eta \). This allows accurate treatment of very sharp jumps in pressure and density.

**Distribution of Birkeland Current.** According to equation (11), magnetic-field-aligned currents occur only where there is a gradient in some \( \eta \), which, in our numerical scheme, occurs only at the inner edge for species \( s \). Thus there is a sheet of Birkeland current flowing to or from the ionosphere along each inner edge, or each computed contour of constant \( \eta \) (see equation [A27] of Harel et al. (1981a)).

**Current Conservation of the Ionosphere.** We convert the differential equation (18) to the following difference equation:

\[
v_{ij} = c_1 v_{i+1,j} + c_2 v_{i-1,j} + v_{i,j+1} + c_4 v_{i,j-1} + c_5 v_{i,j} \tag{22}
\]

where the coefficients \( c_1 - c_4 \) are given by equation (6) of Jaggi and Wolf (1973), and \( c_5 \), which represents the effect of Birkeland currents, is specified by equations (A29) and (A30) of Harel et al. (1981a). Equation (22) is solved by a successive over-relaxation method using

\[
\delta v_{ij} = \frac{1}{6} \left( c_1 v_{i+1,j} + c_2 v_{i-1,j} + v_{i,j+1} + c_4 v_{i,j-1} + c_5 v_{i,j} \right) - v_{ij} \tag{23a}
\]

The improved estimate of \( v_{ij} \) is given by

\[
v_{ij} = v_{ij} + \delta v_{ij} \tag{23b}
\]

The procedure is repeated until the calculated potential changes are sufficiently small that

\[
\sum_{i,j} | \delta v_{ij} |^2 < \epsilon \tag{24}
\]

Actually, the method should perhaps be called successive under-relaxation, because we find that the weighting factor \( \frac{1}{6} \) has to be less than 1 for convergence, usually about 0.5. This simple procedure is adequately efficient for our situation, because we only have about 600 grid points, and we usually have an excellent initial guess for \( \nu \) based on the potential distributions previously calculated for the last two time steps. Usually, variations from one time step to the next are relatively small, and the system converges in \( \sim 20 \) iterations.

**Mapping from the Ionosphere to the Equatorial Plane.** For our recent simulations of the March 22, 1979 magnetic storm, G.-H. Voigt supplied us with magnetic-field models for 16 key times during the event. For each of these models, field lines were traced from each of our ionospheric grid points to the equatorial plane. Results were
entered into our program in the form of matrices specifying the position of the equatorial crossing point, the equatorial magnetic field strength, and the flux-tube volume for each grid point. To evaluate these parameters for arbitrary times during the event, we linearly interpolated between the key-time values stored in the matrices.

**Computation of Boundary Motions.** In the present version of the program, each inner edge is represented by a series of "test particles" (see appendix of Wolf et al. [1982]). The gradient/curvature-drift velocity of each test particle is computed by simple central differences and interpolations, using the value of flux-tube volume and equatorial magnetic field at the grid points. The $E \times B$ drift is treated more carefully because of the fine structure in the computed electric fields in the inner-edge region. The first step is to compute the electric field by straightforward central differences and interpolations applied to the potentials. Then we compute a fine-structure correction by calculating the electric field due to each nearby inner edge. This fine-structure correction compensates for the coarse grid, at least with respect to the electric-field effects of one species' inner edge on another (for a more detailed discussion, see Appendix 2 of Harel et al. [1981a]).

The particles move in time according to equation (21). The time step $\Delta t$ is limited mainly by a numerical instability that occurs when the steplength exceeds approximately the physical decay time of the shortest-wavelength inner-edge ripple that our grid spacing allows us to represent. The very simple step procedure (21) is used because we have not found an alternative that deals more efficiently with the numerical instability and is also simple to adapt to our overall scheme.

**Accuracy Tests.** As in most computer simulations in physics, there are never enough precise numerical-accuracy tests to make one completely sure what the numerical accuracy is for a given physical situation. However, the following tests have provided some indications of accuracy levels:

1. For the case of a single inner edge, uniform ionospheric conductance, dipole magnetic field, and a boundary potential of the simple form (19), time-dependent analytic solutions can be constructed. (Such analytic solutions are helpful for physical discussions [e.g., Siscoe, 1982; Senior and Blanc, 1983].) In the initial testing of our code, we ran a series of comparisons with such analytic solutions, and found good agreement.

2. We have repeated runs several times for different values of the convergence parameter $\varepsilon$ (see equation [24]). Based on those results, we have generally chosen $\varepsilon=1000$ (volts)$^2$ as the best compromise between speed and accuracy.

3. We have repeated runs with decreased values of the time step $\Delta t$, and found very small differences. As a practical matter, if the time step is short enough to avoid the numerical instability
discussed earlier, it seems to give adequate accuracy.

4. The accuracy of the particle-moving algorithm can be tested by running the model with steady input parameters for a time long compared to a drift time, and comparing the computed inner edges with computed contours of constant effective potential

\[ V_{\text{eff}} = V_m + \lambda S \]  \hspace{1cm} (25)

Such a test can be performed for a fast-drifting species (i.e., high \( \lambda \)) in a few hours magnetosphere time. Agreement in such tests has been satisfactory, although problems can occur near local noon, where inner-edge shapes are particularly complex.

Figure 5. Conservation-of-energy test. applied to our simulation of the July 29, 1977 storm. The solid curve gives the rate at which energy is fed into the ring current, by means of currents from the ionosphere and currents through the boundary. The dashed curve gives the rate of change of ring-current energy in the model.

5. For the case where the input magnetic field model remains constant in time, there is a conservation-of-energy check on the system. With no particle loss, the rate of change of particle energy in the modeling region equals the rate at which energy is fed into the particles by Birkeland currents from the ionosphere and by currents and particles flowing through the boundary of the
calculation. Figure 5 shows this energy comparison for the case of the main phase of the July 29, 1977 storm. We feel that this level of error is about as low as we can get it with the present coarse-grid version of the program. Much higher accuracy should be possible with a fine-grid version.

Cost. The advantage of using a coarse grid is, of course, that it allows us to do a lot of runs at reasonable cost. The present version of our main program requires only about 500 kbytes of core, and the CPU cost of running it at night amounts to only about $10 per hour magnetosphere time.

5. REVIEW OF RESULTS

The purpose of this paper has been to discuss the physical basis of the Rice Convection Model, the relationship between the model and standard MHD, and the numerical methods used. We have not discussed the Earth's magnetosphere, only how we model it. In other papers, we have used the model to explain and clarify aspects of the physics of the Earth's magnetosphere. Rather than select one or two such aspects for detailed discussion, we instead present a catalogue to illustrate the variety of topics that the model addresses. Specifically, we list points of magnetospheric physics that have been illuminated through use of the RCM, and indicate where these points are discussed in more detail.

Magnetospheric and Ionospheric Electric Fields

1. Shielding Efficiency. Using idealized analytic models and order-of-magnitude arguments, Vasyliunas (1972) and others have shown that the inner edge of the plasma sheet tends to shield the region earthward of it from the magnetospheric-convection electric field. The RCM has strengthened the theoretical argument for the shielding process, by showing it to be a prominent and persistent feature of much more realistic model calculations (see, e.g., Jaggi and Wolf, 1973; Harel et al., 1981b; Spiro et al., 1981).

2. Shielding Times. The RCM has provided estimates of the time scales for the shielding process, for realistic conditions where ionospheric conductance varies substantially along the shielding layer (Jaggi and Wolf, 1973).

3. Electric-Field Pattern at Subauroral Latitudes During Substorms. With its complicated conductivity model and detailed simulation of ring-current dynamics, the RCM makes predictions as to how the shielding process is disrupted during substorms, allowing magnetospheric convection electric fields to penetrate to lower ionospheric latitudes. The predictions agree well with the observations in most respects (see Spiro et al., [1981] or Maynard et
4. **Dawn-Dusk Convection Asymmetry.** Observations indicate that sunward flow tends to be stronger in the dusk side auroral zone that on the dawn side (Kelley, 1976; Foster, 1983). (This asymmetry is a standard feature of our model calculations, where it results mainly from the change in Hall conductance at the terminators [Harel et al., 1981b].)

5. **Rapid Subauroral Flow.** Very large poleward electric fields are frequently observed over a narrow latitude band just equatorward of the evening auroral zone (Spiro et al., 1974; Heelis et al., 1976; Smiddy et al., 1977; Maynard, 1978; Spiro et al., 1978). Using results from the RCM and related analytic arguments, these events are interpreted as being due to the inner edge of the ion plasma sheet being slightly equatorward of the electron inner edge on the dusk side (Southwood and Wolf, 1978; Harel et al., 1981b).

6. **Effects of Sudden Commencement on Shielding.** From results of our simulation of the magnetic storm of July 29, 1977, which involved a massive compression of the magnetosphere, we predict that the compression of the magnetosphere by increased solar wind dynamic pressure temporarily disrupts shielding and causes substantial sunward flow at subauroral latitudes. To our knowledge, this prediction has not been checked directly in mid- and low-latitude electric-field data.

**Electric Currents**

1. **Overall Pattern of Ionospheric and Birkeland Current.** The overall magnetosphere-driven current system, with region-1 and region-2 Birkeland currents, eastward and westward electrojets, and large meridional Pedersen currents, are intrinsic in our simulations (Wolf, 1974; Harel et al., 1981b; Karty et al., 1982). The same basic pattern emerges from detailed model analysis of ground magnetograms (e.g., Hughes and Rostoker, 1977; Kamide et al., 1981), and other models (e.g., Kamide and Matsushita, 1979).

2. **Nightside-Overlap Region in Birkeland-Current Pattern.** The characteristic down-up-down Birkeland-current pattern observed near midnight (Iijima and Poterma, 1978) usually occurs in our models, where it results from high-energy plasma-sheet ions having inner edges that are shaped differently from low-energy ions and electrons (Wolf et al., 1982; Spiro and Wolf, 1983).

3. **Dawn-Dusk Asymmetry in Low-Latitude AH.** It has long been known that the horizontal component of the low-latitude magnetic field decreases more on the dusk side than the dawn side during a substorm or early in the main phase of a storm. This observation was usually interpreted in terms of a partial ring current in the inner...
magnetosphere, connected through Birkeland currents to the eastward electrojet (e.g., Kamide and Fukushima, 1972; Crooker and McPherron, 1972). When the large-scale Birkeland-current patterns were first discovered (Zmuda and Armstrong, 1974), it was not clear how those patterns related to the idea of a dusk-side partial ring current. Since the complicated current patterns computed in the RCM are consistent with both the observed large-scale Birkeland currents and with the classic dawn-dusk asymmetry observed in low-latitude magnetograms, we have been able to interrogate the model to determine which currents are responsible for which observational features. This exercise resulted in a modification of the dusk side partial-ring current picture of the substorm current system (Harel et al., 1981b; Chen et al., 1982). Using hints from the computer results, Crooker and Siscoe (1981) have constructed an elegant analytic theory of this same asymmetry.

4. *Birkeland Current Patterns Just After Strong Magnetospheric Compressions.*

Our simulations of the July 29, 1977 storm indicate that the general pattern of region-1/region-2 Birkeland current may be temporarily and dramatically disrupted during a strong magnetospheric compression. As far as we know, this theoretical prediction has not been tested observationally.


Region-1 currents have frequently been observed on field lines that are drifting slowly toward the Sun (e.g., Smiddy et al., 1980; Mozer et al., 1980). Model results and related analytic calculations indicate that such region-1 currents are a natural result of gradients in particle content among different plasma-sheet flux tubes (Karty et al., 1983).

**Ring-Current Injection**

1. *Convection and Ring-Current Injection.* It was suggested many years ago that enhanced convection during a magnetic storm injected some plasma-sheet plasma deep into the magnetosphere to form the storm-time ring current. This idea was supported by particle-trace calculations done with simple models of the magnetospheric electric field (see review by Kivelson et al. [1979]). Our simulations of the magnetic storms of July 29, 1977 and March 22, 1979 have shown that enhanced convection could inject plasma-sheet plasma to form a storm-time ring current, even considering that the injection is inhibited by the shielding effect, which we attempt to calculate realistically with the RCM (Wolf et al., 1982; Spiro and Wolf, 1983).

2. *Relationship Between Joule Heating and Convection.* The time-integrated Joule heating of the upper atmosphere through a substorm or storm main phase is, in our simulations, typically of the same order of magnitude as the change in ring-current energy. In events where the injection is entirely due to enhanced convection, the integrated Joule heating is larger. In events where the injection is
due in large part to magnetospheric compression. integrated Joule heating tends to be smaller than the ring current energy (Harel et al., 1981b; Wolf et al., 1982). The theoretical necessity for integrated Joule heating to be of the same order of magnitude as the ring-current increase in an injection event has now been derived by two analytic arguments (Siscoe, 1982; Harel et al., 1981b).

3. Factors Affecting Ring-Current Injection. A recent long series of computer experiments, done partly in collaboration with George Siscoe, has clarified a number of cause-and-effect relationships governing ring-current injection (Spiro and Wolf, 1983). The following results hold within the physics presently included in our model:

(i) Increasing ionospheric conductance increases the energy of the injected ring current, though not dramatically. (Increasing conductance decreases shielding efficiency.)
(ii) Compression of the magnetosphere plays an important role in injection of the ring current.
(iii) Decreasing the pressure in the plasma sheet but keeping temperature constant causes ring-current plasma to be injected deeper into the magnetosphere (weaker shielding), but decreases the total strength of the ring current (less plasma available to inject).
(iv) Increasing the temperature of the plasma-sheet ions slightly decreases total ring-current energy, but allows deeper injection.

Convection and Magnetic Field Configuration

A research effort motivated by the need to obtain reliable plasma boundary values for the RCM showed that there is an inconsistency between the idea of steady, subsonic, sunward convection in the plasma sheet, and the requirement that the magnetic-field configuration be closed and in stress balance (Erickson and Wolf, 1980). This theoretical difficulty, which may, we think, contain the key to substorm occurrence, has been verified and further investigated by Schindler and Birn (1982) and Tsyganenko (1982).

6. CONCLUDING COMMENTS

The Rice Convection Model has provided substantial physical insights into how convection works in the Earth's inner magnetosphere. We are now close to the point where the major cause-and-effect relations are established within the physics presently included in our model. The following things remain to be done:

1. Develop a version of the program that will deal efficiently with a denser grid, for more spatial detail and higher overall accuracy.
2. Add additional physics to the model, doing many computer experiments to determine the key cause-and-effect relationships involved. The most obvious additional physical effects are the following: particle loss by charge exchange and precipitation; magnetic-field-aligned electric fields; ion injection from the ionosphere; equatorial electrojet; Van Allen-belt particles; and neutral winds.

3. Combine the convection model with a time-dependent magnetic-field model that is consistent with the currents computed in the convection calculation.

The Rice Convection Model has proven to be an effective theoretical tool for investigating the physics of the inner magnetosphere. The continued development of the model to include additional physical processes promises to be just as fruitful.

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REFERENCES

Lyon, J., this volume, 1984.
Wolf, R. A., in B. M. McCormac (ed.) Magnetospheric Physics, D.
Wu, C. C., this volume, 1984.